ISSN 1691-5402
ISBN 978-9984-44-028-6
Environment. Technology. Resources
Proceedings of the 7th International Scientific and Practical Conference. Volume I1

DESCRIPTIVE MODEL OF SLIDING FRICTION PROCESSES
SLĪDES BERZES PROCESU APRAKSTOŠS MODELIS

Andris Martinovs[1], Vladimir Gonca[2]
1- Rezekne Higher Education Institution
Atbrivosanas aleja 90, Rezekne, LV 4600, Latvia
Phone:+371 28325519; fax:+371 4625167; e-mail: andris.martinovs@ru.lv
2- Riga Technical University, Institute of Mechanics
Ezermalas Str. 6 - 305, Riga, LV 1014, Latvia
Phone: +371 7089317; fax: +371 7089748; e-mail: Vladimirs.Gonca@rtu.lv

Abstract: *Paper analyses the sliding friction coefficient of rubber on concrete, timber and ceramic tile surfaces depending on the weight of the sliding object and contact surface area. It has been established that increase in the weight of the object makes sliding friction coefficient to grow. In the case of increase in size of contact area, sliding friction coefficient between rubber and concrete also increases, but it decreases between rubber- timber and rubber- tile. The mathematical model for description of sliding friction process has been developed which can be used to determine optimal surface area and a pattern as well as optimal weight of the sliding object in order to provide sufficient sliding friction. Model has five independent constants. It includes the contact surface area, the weight and the velocity of the sliding object, sliding friction coefficient, temperature and time.*

Key words: *friction, mathematical model, rubber.*

Introduction

In the design of means of conveyance, tyres, wheelchairs etc. sliding has to be eliminated. In order to provide good contact with main surface, materials with high sliding friction coefficient have to be chosen as well as optimal pattern and size of elements which provide grip on surface. These problems are analysed by many authors in their works, for example [1-7]. The objective of the paper is to develop the mathematical model describing sliding friction processes in order to optimize friction affecting parameters.

Materials and methods

In order to develop mathematical model, the initial experimental research has been performed. Rectangular plates of organic glass with size 44× 42× 3 mm are used as a sample. 2 mm thick rectangular rubber is glued to the bottom of the plate and a weight is put on top the plate. In the case of small contact areas instead of the layer of rubber 2 mm thick rubber strips are glued to the corners of the plate. The sample is steadily pulled on horizontal surface at a velocity 5±1 mm/s and force of friction is measured. Sliding friction coefficient is calculated:

$$\mu = \frac{F}{P}, \tag{1}$$

where F – force of friction; P – total weight of weights and the sample. Each sample with the same loading is subjected to 10 measurements on the different locations on concrete, timber and tile surfaces. Materials used: 1) rubber: natural rubber NR- 55,46 %, filler K354- 27,73 %, vulcanisation temperature- 160 ^{0}C, vulcanisation time- 9 minutes, producer - Baltijas gumijas fabrika; 2) concrete; 3) timber plank, dry, planed; 4) ceramic tile.

Experimental results

Sliding friction coefficient depending on pressure loading P on the sliding object for different contact surfaces S between rubber and concrete is shown in Figure 1, between rubber and timber is shown in Figure 2 and between rubber and ceramic tile is shown in Figure 3. There are μ median values, accidental error intervals, approximate function graphs, their mathematical expressions and values of coefficient of determination R^2 shown in Figures.

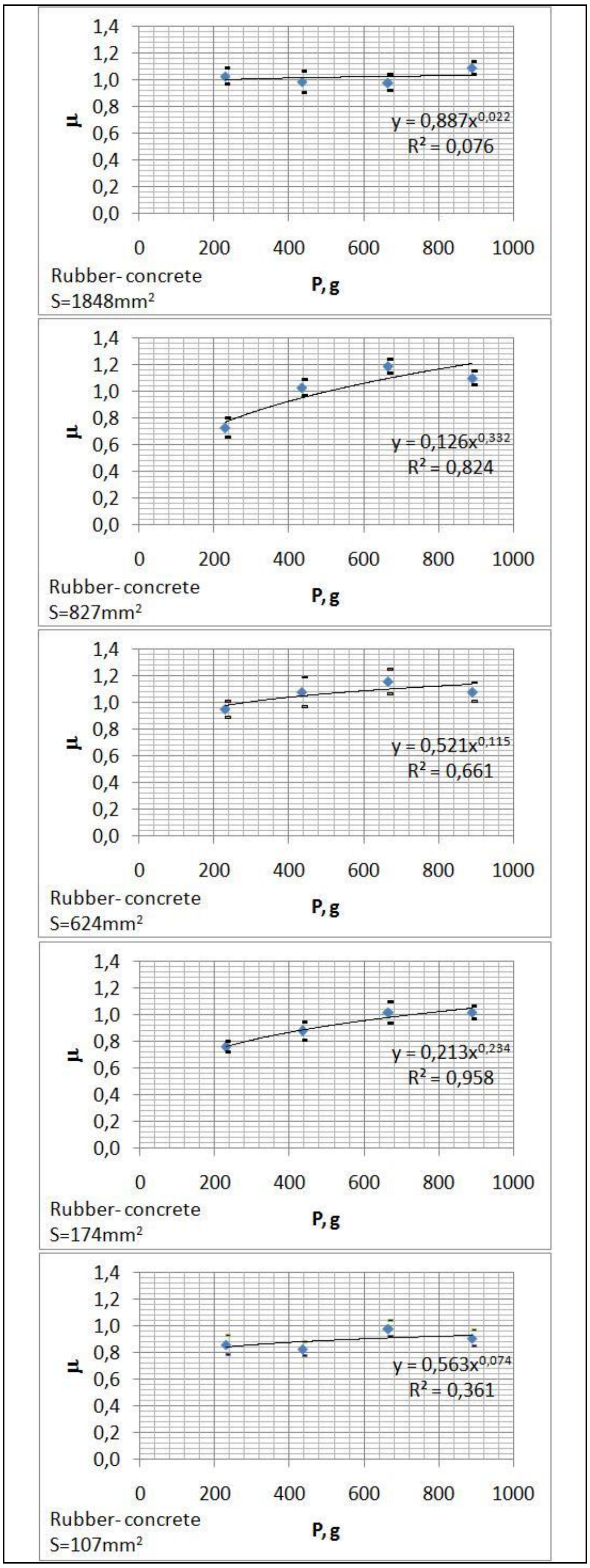

Fig.1. **Dependence of sliding friction coefficient between rubber and concrete on weight for different contact surface areas**

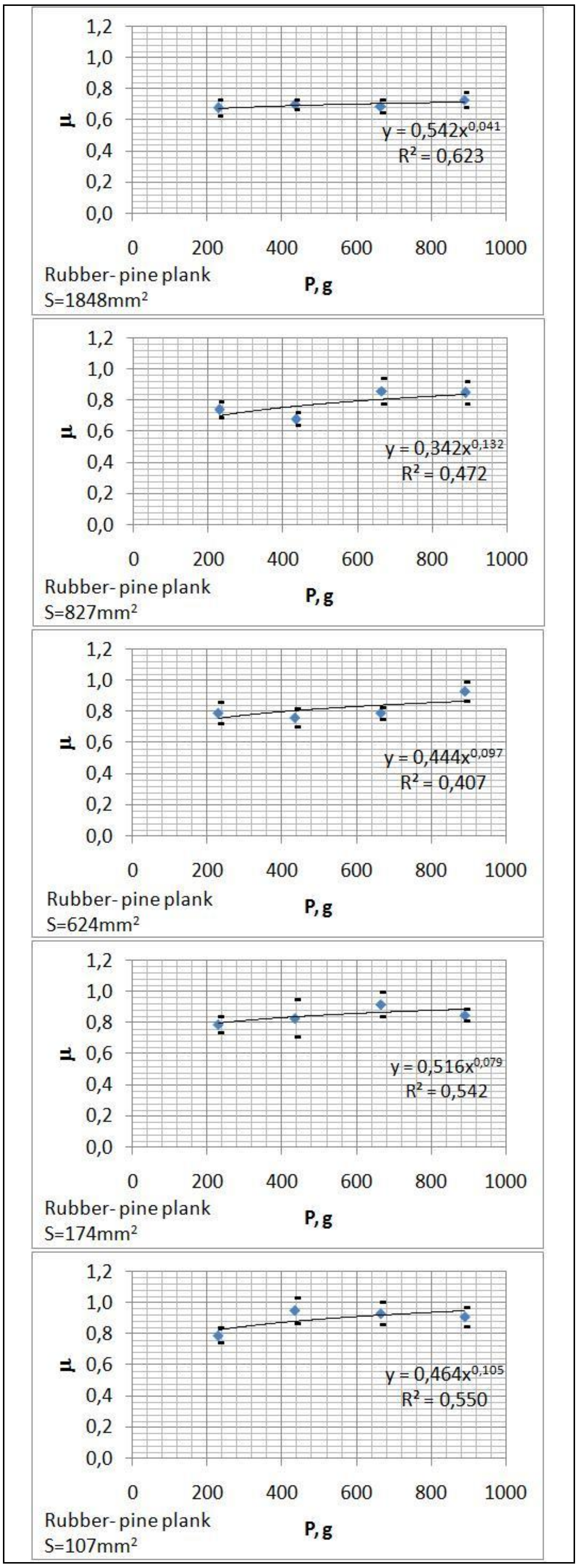

Fig.2. **Dependence of sliding friction coefficient between rubber and pine plank on weight for different contact surface areas**

Examples of the dependence of sliding friction coefficient between rubber and concrete, rubber and timber, rubber and ceramic tile on contact surface areas are shown in Fig. 4 – 6.

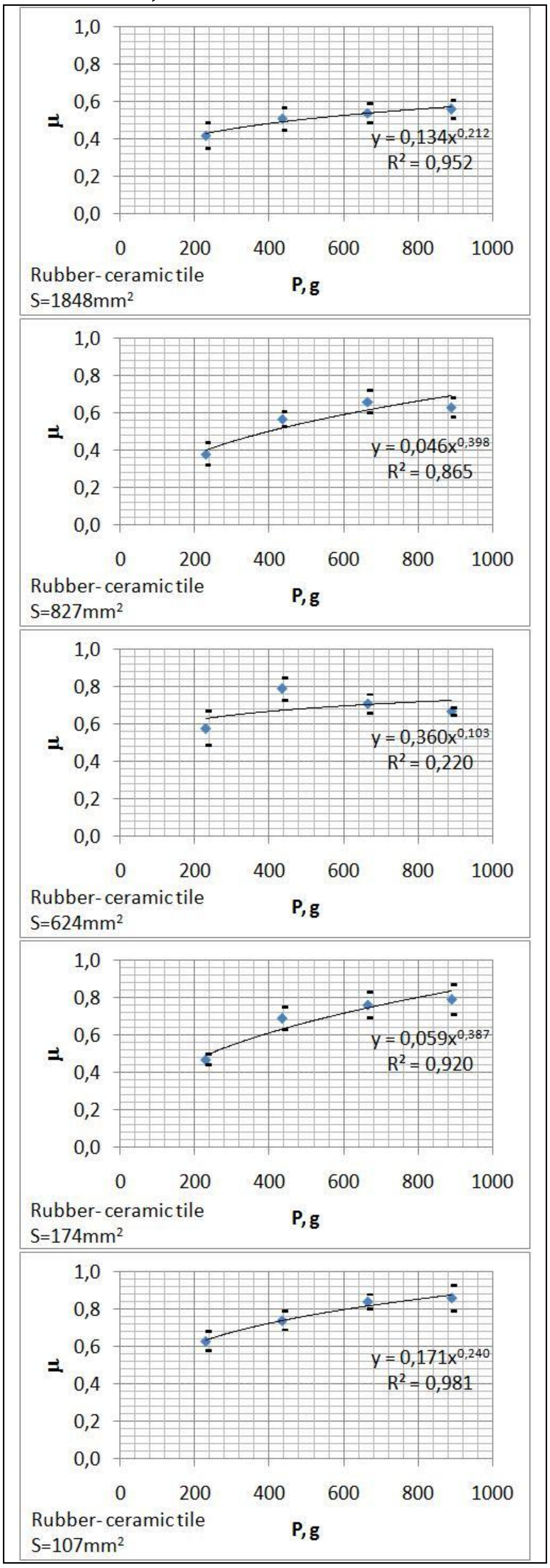

Fig.3. **Dependence of sliding friction coefficient between rubber and ceramic tile on weight for different contact surface areas**

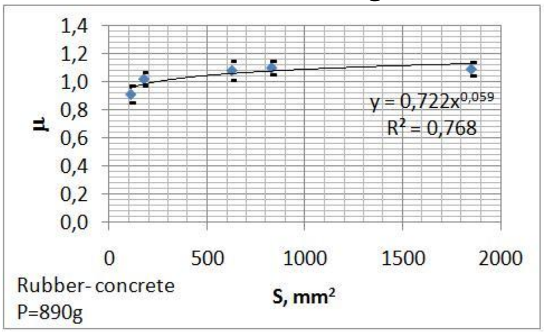

Fig.4. **Dependence of sliding friction coefficient between rubber and concrete on contact surface area**

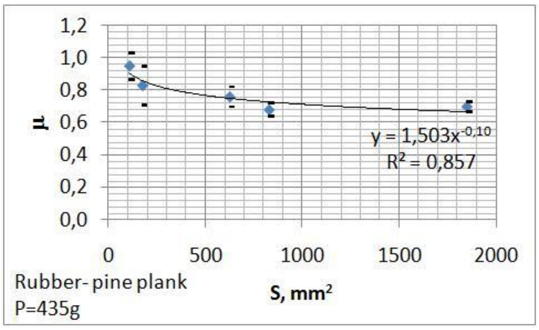

Fig.5. **Dependence of sliding friction coefficient between rubber and pine plank on contact surface area**

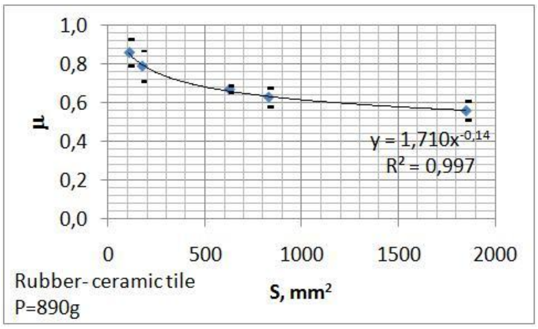

Fig.6. **Dependence of sliding friction coefficient between rubber and ceramic tile on contact surface area**

In the experiments used rubber structure is shown in Figure 7 (got on Scanning Electron Microscope EVO MA15 in Tallinn University of Technology).

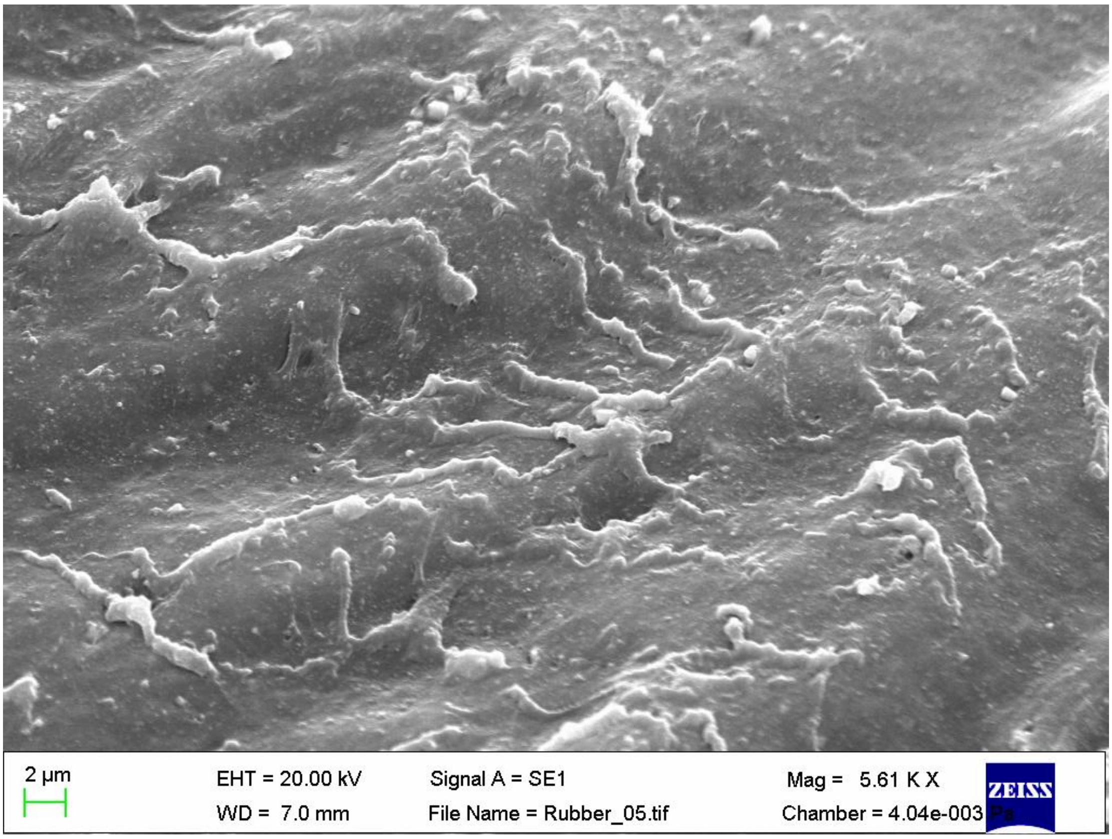

Fig.7. **Structure of rubber which used in the experiments**

Mathematical model

Sliding friction force F is directed on contact area opposite the motion of the sliding object; this force is created by chemical bonds between both objects (see Fig. 8). Friction force

$$F = \mu \cdot P, \tag{2}$$

where μ- sliding friction coefficient, P- the weight of the top object. If P increases, contact surface is distorted, its area becomes larger, more bonds are formed between atoms of both objects and mutual sliding of both objects becomes more difficult. Consequently, if P increases, friction force has to grow.

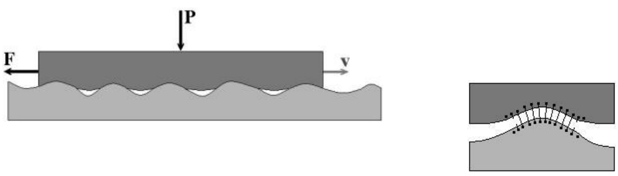

Fig.8. **Interdependent positioning of sliding surfaces and bonds between their atoms**

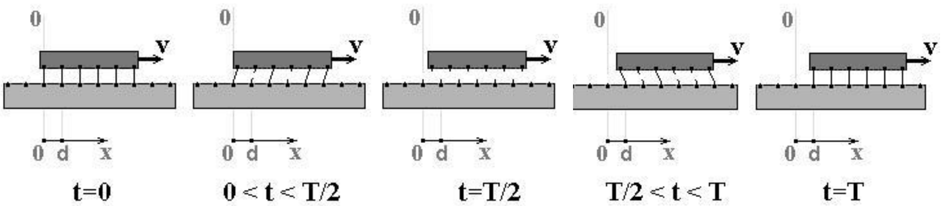

Fig.9. **Rupture and formation of the bonds during process of sliding friction**

Let's replace curved surface of both objects in contact with flat surface (see Fig. 9). Let's assume that: 1) the top object moves on the surface of the bottom object with constant velocity v; 2) atoms of both objects create cubic lattice with lattice constant d; 3) all bonds between both objects have the same energy values Q; 4) at time t=0, the top object coordinate x=0, bonds are not distorted, total number of bonds is N. Moving the top object, bonds are stretched and ruptured. When displacement x reaches value d/2 all bonds are ruptured. (see Fig. 9, moment in time T/2). Next follows the formation of new bonds which try to contract and pull the top object forward. At the moment in time t=T (T- period) bonds are not distorted, the top object has moved for distance d in relation to the bottom object.

The performed experiments show that increase in the weight of the object causes sliding friction coefficient to grow. Increasing displacement value x of the top object (see Fig. 9), bonds are stretched further and the instantaneous value of friction force F grows. Considering the above, let's assume that friction forces

$$F = A \cdot P^n \cdot x^m, \tag{3}$$

where A, n, m- constants; m≥1; if m=1, then bonds are only subjected to elastic distortion, when F~x; n≥1; if n=1, then sliding friction coefficient is constant value, not dependent on P. It derives from equations (2) and (3) that the instantaneous value of sliding friction constant

$$\mu_* = A \cdot P^{n-1} \cdot x^m. \tag{4}$$

The graph, showing changes in the values of this parameter depending on the top object displacement x, is given in Figure 10.

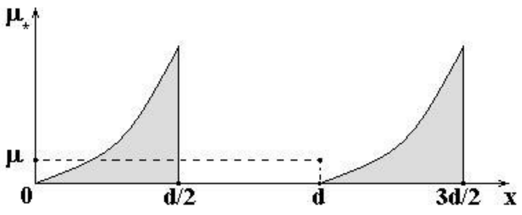

Fig.10. **Instantaneous sliding friction coefficient depending on coordinate x; μ- median sliding friction coefficient**

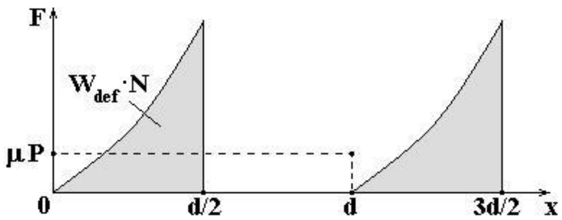

Fig.11. **Sliding friction force depending on coordinate x; μ·P- median sliding friction force**

Median value μ of sliding friction coefficient and instantaneous values μ_* are linked

$$\mu \cdot d = \int_0^{d/2} \mu_* \cdot dx. \tag{5}$$

By putting equation (4) in this expression and doing integration, one has

$$\mu = \frac{A \cdot P^{n-1} \cdot d^m}{(m+1) \cdot 2^{m+1}}. \tag{6}$$

If sliding friction coefficient median values μ_1 and μ_2 at two different loadings P_1 and P_2 are known, constant n can be found from equation above:

$$n = 1 + \log_{\frac{P_1}{P_2}} \frac{\mu_1}{\mu_2}. \tag{7}$$

To determine the number of bonds N at the initial moment t=0, let's use conditions: 1) if P=0, then N=0; both objects are not in contact, there are no bonds between them; 2) if P→∞, then $N \to S/d^2$, where S – area of bottom surface of the top object, d^2 – area taken by one atom on the surface, the top object is pressed to the surface of bearing with such force that all its bottom surface atoms form bonds with atoms of the surface of bearing. These conditions are expressed in function

$$N = \frac{S}{d^2} \cdot \left(1 - e^{-C \cdot P}\right), \tag{8}$$

where C- constant.

Potential deformation energy delivered to one bond W_{def} is equal to total deformation energy of bonds divided by number of bonds N. Total deformation energy of the bonds is equal to work done by friction force. Therefore, considering equation (3),

$$W_{def} = \frac{\int_0^x F \cdot dx}{N} = \frac{\int_0^x A \cdot P^n \cdot x^m \cdot dx}{N} = \frac{A \cdot P^n \cdot x^{m+1}}{N \cdot (m+1)}. \tag{9}$$

Time period T: 1) the top object displacement x=d; 2) considering expression (9), the work of friction force (see Figure 11):

$$\mu \cdot P \cdot d = \frac{A \cdot P^n \cdot \left(\frac{d}{2}\right)^{m+1}}{(m+1)}. \tag{10}$$

The displacement of the top object in a case of steady movement:

$$x = v \cdot t. \tag{11}$$

It derives from equations (8) – (11) that potential deformation energy delivered to one bond

$$W_{def} = \frac{\mu \cdot P \cdot (2 \cdot v \cdot t)^{m+1}}{S \cdot \left(1 - e^{-C \cdot P}\right) \cdot d^{m-2}}. \tag{12}$$

Energy of the thermal motion, delivered to one bond:

$$W_{silt} = k \cdot T_a, \tag{13}$$

where k- Boltzmann constant, T_a- absolute temperature. Changes in number of bonds within time interval dt [8]:

$$dN = -N(t) \cdot Z \cdot f \cdot e^{-\frac{Q}{W_{silt} + W_{def}}} \cdot dt, \tag{14}$$

where N(t) – number of bonds at the moment in time t; Z=6 – coordination number of cubic lattice; $f = 1 \cdot 10^{13}$ Hz – oscillation frequency of atomic thermal motion; Q - bond energy; exponent describes probability of appearance of energetic fluctuation that would be sufficient for bond rupture. Considering potential deformation energy delivered to one bond and expressions of thermal motion energy (12) and (13), changes in number of bonds can be expressed:

$$dN = -N(t) \cdot Z \cdot f \cdot e^{-\frac{Q}{k \cdot T_a + \frac{\mu \cdot P \cdot (2 \cdot v \cdot t)^{m+1}}{S \cdot \left(1 - e^{-C \cdot P}\right) d^{m-2}}}} \cdot dt. \tag{15}$$

Initial and final conditions for the solution of differential equation (15):

if t=0, then $N(0) = \frac{S}{d^2} \cdot \left(1 - e^{-C \cdot P}\right)$; (16)

if t=T/2, then N(T/2)=0. (17)

Consequently, the mathematical model describing friction processes is created. It has five constants C, d, n, m, Q. The model includes contact surface area S, the weight of the top object P as well as velocity of movement v, sliding friction coefficient μ, temperature T_a and time t.

Examples of use of the model

The model can be used to find optimal contact surface area S with given P and known μ. With equation (15) with different values for S velocity of the top object v is calculated. Optimal S value is one which matches the minimal velocity v (it means that the greatest friction is achieved).

It is possible to find optimal pressure force P with given area S and known relation μ=μ(P). With equation (15) with different values for P velocity of the top object v is calculated. It derives from (15) that the greatest sliding friction (v→0) will be reached if P→∞. Infiniti force can not be considered optimal. Therefore force P, at which velocity preset limit, e.g. 0.1mm/s, is reached, can be considered optimal.

The model can be used to optimise a pattern of tyre tread. In this case 1) contact surface has to be divided in small elements ΔS; 2) mechanical tension field for each ΔS has to be calculated, e.g. with multiphysics modeling software Comsol; 3) median pressure force P on each element has to be calculated; 4) with equation (15) sliding velocity v for each element ΔS is found. As all area elements ΔS are connected and do not mutually move, it can be assumed that sliding will start when element with least velocity will exceed critical velocity value, e.g. 0.1 mm/s. Tyre tread has optimal pattern in case when minimal velocity values v are obtained.

Conclusion

The dependence of sliding friction coefficient on pressure force for contact surfaces areas between rubber and concrete, timber as well as ceramic tile is investigated.

The mathematical model, describing sliding friction process that allows searching for the optimal values of contact areas and weight, as well as searching for the optimal pattern of contact surface in order to provide the maximum sliding friction, is developed.

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