Abstract—The article discusses methods for constructing piston outer profile for the rotary type expansion machine in order to reduce unwanted (parasitic) volumes and offers options for constructing outer profile of piston using analytical and geometric methods.

Keywords—lever-cam motion converter, power generation system, rotary engine.

I. INTRODUCTION

The subject of this article is the rotary expansion machine (patent RU2619391) published by the authors in 2017 [1].

The machine contains four pistons, forming an equilateral four-link mechanism in the form of a mobile rhomboid. The ends of pistons are in full sliding contact with inner surface of chamber, the profile of which is the external equidistant, spaced by a distance of radius \( R_1 \) of ends of piston from reference curve, described in polar coordinates \( \rho, \alpha \) with equation

\[
\rho(\alpha) = L \sin \left( a + b \cos 2\alpha \right),
\]

where \( L \) is the distance between the axes of the piston hinges; \( a = \frac{\pi}{4}; b = \frac{\pi}{4} - \psi_{\min} / 2 \), and where \( \psi_{\min} \) is the minimum angle between the pistons.

The optimal value of this angle is \( \psi_{\min} = 1.221 \) rad, at which \( a = 0.7854 \) rad, \( b = 0.1749 \) rad. The outer surface of each piston has the shape of an arc of a circle of radius \( R \), the value of which is chosen in such a way that in the position \( \alpha = \frac{\pi}{4} = 45^\circ \), in which the four-link mechanism is square, the surface of piston touches internal cavity of chamber at three points. Therefore, in this position, two working volumes are formed, isolated from the rest of chamber space.

The task is to determine outer surface of the piston, in which in the position \( \alpha = \frac{\pi}{4} \), the working volumes will be absent, i.e. piston profile will coincide completely with inner surface of chamber, which is equidistant.

II. FIRST OPTION

The initial position of piston AB with \( \alpha = \frac{\pi}{4} \) is shown in Fig. 1.

Fig. 1. Initial position of piston

Point E belongs to equidistant, and point M coinciding with it belongs to piston. We introduce a fixed coordinate system \( xOy \) with the beginning at the point O of the worker in the center of the cylinder. We introduce the following notation: \( AB = L, OC = L/2, y_E = L \sin(a+b)+R_1 \). When \( a = 0.7854, b = 0.1749 \), we get \( y_E = 0.573L + R_1 \).

Then the segment \( CE = h_E = y_E - 0.5L \), or \( h_E = 0.079L + R_1 \).

Fasten with the piston the moving coordinate system \( C\xi\eta \) with the origin at point C (Fig. 2).

Fig. 2. Arbitrary position of piston

In moving axes, point M will have coordinates \( \xi_M, \eta_M \). Determine the trajectory of point M when the piston moves from the position \( \alpha = \frac{\pi}{4} \). That the angle

Print ISSN 1691-5402
Online ISSN 2256-070X
http://dx.doi.org/10.17770/etr2019vol3.4059
Published by Rezekne Academy of Technologies.
This is an open access article under the Creative Commons Attribution 4.0 International License.
a decreases by the value determined by the angle γ, i.e. α = π/4 − γ. Then the coordinates of the point M are defined as

\[
\begin{align*}
\xi_M &= x_c + \xi_u \cos(x, \xi) + \eta_u \cos(y, \eta) = \frac{L}{2} \cos \varphi_1 + h_\xi \cos \varphi_1, \\
y_M &= y_c + \xi_u \cos(y, \xi) + \eta_u \cos(y, \eta) = \frac{L}{2} \sin \varphi_1 - h_\eta \cos \varphi_1,
\end{align*}
\]

where ξ_1 is the angle between axes ηx; η_1 is the angle between axes ξx.

Express the angles φ_1 and φ_2 through the angle γ:

\[
\begin{align*}
\varphi_1 &= \pi - \gamma \quad \varphi_2 = 2\pi - \gamma.
\end{align*}
\]

Then the trajectory of the point M in a parametric form is

\[
x_M = x_c + \xi_u \cos(x, \xi) + \eta_u \cos(y, \eta) = \frac{L}{2} \cos \varphi_1 + h_\xi \cos \varphi_1, \\
y_M = y_c + \xi_u \cos(y, \xi) + \eta_u \cos(y, \eta) = \frac{L}{2} \sin \varphi_1 - h_\eta \cos \varphi_1.
\]

Solving the system of equations (4) with respect to ξ_M, η_M and taking into account that \(x_c = L/2 \cos \varphi, y_c = L/2 \sin \varphi\), \(\varphi_1 = \varphi/2\), \(\varphi_2 = \pi/2\) we get

\[
\begin{align*}
\xi_M(t) &= \rho(\alpha)(t) = \frac{L}{2} \cos(2b \sin 2\gamma(t)) + y_M \sin(\gamma), \\
y_M(t) &= \frac{L}{2} \sin(2b \sin 2\gamma(t)) + y_M \cos(\gamma).
\end{align*}
\]

The solution of this system of equations has a parametric form:

\[
\begin{align*}
\xi(t) &= \xi(t)(\gamma), \\
y(t) &= \eta(t)(\gamma).
\end{align*}
\]

In the position \(\alpha = \pi/4 - \gamma\) the point M of the piston must coincide with the point E of the equidistant chamber in the position \(\alpha = 45^\circ\). This will jam when moving from this position. Obviously, the profile synthesis problem is multivariate. Second option

Computer simulation made in SolidWorks has shown that if the piston profile is given exactly the profile of the equidistant chamber in the position \(\alpha = 45^\circ\). This will jam when moving from this position. Obviously, the profile synthesis problem is multivariate. Second option

Fig. 3 shows the piston in an arbitrary position.

The current position of the piston AB in the fixed coordinate system Ox.y is determined by the coordinates \(x_A, y_A\) of the hinge A, as well as by the angle γ, of the inclination of the axis of the piston AB to the axis Ox.

\[
\begin{align*}
x_A &= x_c + \xi_M \cos(x, \xi) + \eta_M \cos(y, \eta) \\
y_A &= y_c + \xi_M \cos(y, \xi) + \eta_M \cos(y, \eta).
\end{align*}
\]

Where: \(x_A = \rho_{\alpha} \cos \varphi, y_A = \rho_{\alpha} \sin \varphi\), \(\rho_{\alpha} = \rho(\alpha) = L \sin(\alpha + \frac{3\pi}{4} + \cos 2\alpha)\), \(\varphi_1 = \frac{\pi}{4} - \frac{3\pi}{4} \cos 2\alpha\).

Let us introduce the moving coordinate system \(\xi\eta\) fixed to the piston with the beginning at point A. We choose an arbitrary point M on the piston profile, which has coordinates \(\xi_M, \eta_M\) in the moving coordinate system \(\xi\eta\). The coordinates of this point in the fixed coordinate system Ox.y are determined by the coordinate transformation formulas:

\[
\begin{align*}
x_M &= x_A + \xi_M \cos(x, \xi) + \eta_M \cos(y, \eta) \\
y_M &= y_A + \xi_M \cos(y, \xi) + \eta_M \cos(y, \eta).
\end{align*}
\]

Consider the same piston in the position \(\alpha = 45^\circ\) (Fig. 4)

Fig. 4. Position of the piston at \(\alpha = 45^\circ\)

In this position, the three points of the piston profile, \(D_1, D_2\) touch the equidistants of the chamber at points E, E1, E2. The highest point of the piston profile D has the following coordinates:

\[
\begin{align*}
x_D &= x_E = 0, \\
y_D &= y_E = \rho(\alpha) + R_1 = L \sin(\alpha + b) + R_1.
\end{align*}
\]
When the piston moves from the position to the position $\alpha = 45^\circ + \Delta \alpha$, the points $D_1$, $E_1$ and $D_2$, $E_2$ change their position, but the contact between them remains. The upper point of the piston $D$ will also leave the point of the chamber $E$. We will find such a profile of the piston, at which the contact of the points of the profile $M$ and the point $E$ of the chamber will remain in a certain range of the increment of the angle $\Delta \alpha$. This means that it is necessary to determine the coordinates $\xi_M$ and $\eta_M$ of the point $M$ so that in the position $\alpha = 45^\circ + \Delta \alpha$, the coordinates of point $x_M$ and $y_M$ are equal to the coordinates $x_E$ and $y_E$ of point $E$.

$$x_M(45^\circ + \alpha) = x_E,$$
$$y_M(45^\circ + \alpha) = y_E = L \sin(\alpha - b) + R_1.$$  \hspace{1cm} (9)

As a result, we arrive at the following system of equations for $\xi_M$ and $\eta_M$:

$$\begin{cases} \xi_M \sin \varphi_1(\beta) + \eta_M \cos \varphi_1(\beta) = x_A(\beta) \\ \xi_M \cos \varphi_1(\beta) + \eta_M \sin \varphi_1(\beta) = y_A(\beta) + L \sin(\alpha - b) + R_1, \end{cases}$$

where $\beta = 45^\circ + \alpha$, $x_A(\beta) = \rho_A(\beta) \cos \beta$, $y_A(\beta) = \rho_A(\beta) \sin \beta$, $\rho_A(\beta) = L \sin(a + b \cos 2\beta)$, $\varphi_1(\beta) = \beta + \frac{3\pi}{4} + b \sin 2\beta$.

Solving system (10) according to Kramer’s rule we get:

$$\xi_M = \begin{vmatrix} \sin \varphi_1(\beta) & \cos \varphi_1(\beta) \\ \cos \varphi_1(\beta) & \sin \varphi_1(\beta) \end{vmatrix}^{-1} = \begin{vmatrix} x_A(\beta) & \cos \varphi_1(\beta) \\ y_A(\beta) + L \sin(\alpha - b) + R_1 & \sin \varphi_1(\beta) \end{vmatrix},$$
\hspace{1cm} (11)

$$\eta_M = \begin{vmatrix} \sin \varphi_1(\beta) & x_A(\beta) \\ \cos \varphi_1(\beta) & y_A(\beta) + L \sin(\alpha - b) + R_1 \end{vmatrix}.$$ (12)

Then

$$\xi_M = \frac{x_A(\beta) \sin \varphi_1(\beta)}{y_A(\beta) + L \sin(\alpha - b) + R_1 \cos \varphi_1(\beta)},$$
$$\eta_M = \frac{y_A(\beta) \cos \varphi_1(\beta) + L \sin(\alpha - b) + R_1 \sin \varphi_1(\beta)}{y_A(\beta) + L \sin(\alpha - b) + R_1 \sin \varphi_1(\beta)}.$$ (12)

Varying the value of the increment of the angle $\alpha$ in the range $0 \leq \alpha \leq 45^\circ$, or the same as the angle $\beta = 45^\circ + \alpha$ in the range $45^\circ \leq \alpha \leq 90^\circ$, from equation (12) we obtain the desired piston profile.

In the process of movement each point $M(\xi_M, \eta_M)$ of the synthesized piston profile will move along its own trajectory of the form $x_M = x_M(\alpha)$ and $y_M = y_M(\alpha)$, defined by equation (7). It is important to check the absence of the intersection of these trajectories with the equidistant curve defined by the equations $x_E = x_E(\alpha)$ and $y_E = y_E(\alpha)$. Otherwise, the piston will jam in the housing.

If there is an intersection of the trajectories with equidistant, the wording of the problem should be changed. We require that in the position $\alpha = 45^\circ + \alpha$, the coordinates $x_M$ and $y_M$ of the point $M$ should be equal to the coordinates of $x_E'$ and $y_E'$ of point $E'$ (Fig. 5, a) located on the axis $Oy$ below point $E$ at some distance $= (\alpha)$. Then we have

$$x_M(45^\circ + \alpha) = x_E',$$
$$y_M(45^\circ + \alpha) = y_E' = y_E = L \sin(\alpha - b) + R_1.$$ (a).

Fig. 5. Types of functions

In this case equation (12) is somewhat modified and takes the form

$$\begin{cases} \xi_M = \frac{x_A(\beta) \sin \varphi_1(\beta)}{y_A(\beta) + L \sin(\alpha - b) + R_1 \cos \varphi_1(\beta)}, \\ \eta_M = \frac{y_A(\beta) \cos \varphi_1(\beta) + L \sin(\alpha - b) + R_1 \sin \varphi_1(\beta)}{y_A(\beta) + L \sin(\alpha - b) + R_1 \sin \varphi_1(\beta)}. \end{cases}$$

The type of the function $h(\Delta \alpha)$, which varies in the range $0 \leq \Delta \alpha \leq 45^\circ$, should be specified. It is known that this function should be equal to zero at two extreme points $\Delta \alpha = 0$ and $\Delta \alpha = 45^\circ$. It can be offered two types of this function:

1. sinusoidal (Fig. 5, b): $\alpha = \frac{\max}{\sin(4(\alpha))} (a)$ cosine (Fig. 5, c): $\alpha = \frac{\max}{\cos(\alpha))}$. (a).

In both cases it is possible by varying the value of $h_{\max}$ to achieve the absence of jamming of the piston in the housing. The decision on which option is preferable can be made based on the results of calculations.

REFERENCES


Zhuravlev et al. Construction of Piston Outer Profile for Rotary Type Expansion Machine


