# THE CHOSEN CLASSICAL AND NON-CLASSICAL ALGORITHMS IN THE GEODETIC MEASUREMENTS OF DISPLACEMENTS

## *KLASISKO UN NEKLASISKO ALGORITMU IZVĒLE DEFORMĀCIJU ĢEODĒZISKAJOS MĒRĪJUMOS*

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**Abstract.** *The approach to analysing the measurement results of a levelling network which was established to identify the vertical displacements of the earth dam crone of a waste pond was presented in the paper. The obtained measurement results carried out by a precise levelling method were used in determining the precise displacements of the controlled points in the time function. The non-linear models of a kinematic network were applied to describe the displacements. A numerical evaluation of periodic measurement results was carried out by classical algorithms and neural networks. The neural networks recognized as the sophisticated modelling techniques were able to copy the non-linear functions.*

**Keywords:** *non-uniform settlements, kinematic geodetic network, neural networks.*

### Introduction

The kinematic state of a geodetic measuring-controlling network is determined on the basis of the estimated parameters values of a given kinematic model. This model describes the points movement. The kinematic model of a geodetic network differs from the static model because it also enables to record the object deformation changes in time and in space. The following three issues have been taken into consideration while analysing the measurement results in the kinematic way: determination of a hypothetical model structure, estimation of numerical values, checking the model adequacy (its structure and the estimated values).

### The characteristics of a given object

The waste pond, consisting of two chambers, was located on the area of an industrial establishment. The sketch of a geodetic network consisting of 30 measurement points located on the earth dam crone of the waste pond was presented in the fig.1. The points were located according to the project made by the experts in geotechnics. The measurement periods were carried out within the years 2001-2004. Calculations were done by the precise levelling method by the means of a levelling instrument Ni 007 and at the same time the 70 height differences were observed in each measurement period.

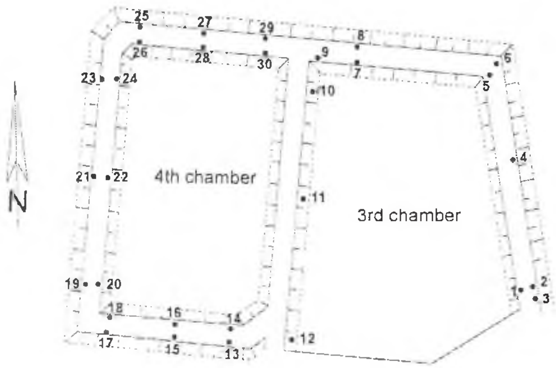


*Fig.1.* **The sketch of the points arrangement of a measuring-controlling network**

### The data analysis

The displacements research was preceded by the data analysis. The simplest protection was applied to avoid observation errors which had the values severally crossing the limit of random errors in the form of the loop traverse closing. The error value $m_{[\Delta h]} = 2m\sqrt{n+n'}$ was assumed as the correctness criterion of the done observations. The value $m$ means a measurement square error of a single height difference and the values $n$ and $n'$ mean the number of the levelling instrument stations in the „forward" and „backward" direction. The precise control was carried out on the basis of the correction number ratio to its square mean error:

$$\frac{|v|}{m_v} < 2 \tag{1}$$

The diverged observations $(|v|/m_v) > 2$ were eliminated on the basis of the observation adjustment results done by a least square method within the minimal limitation of the freedom degrees.

## The sensitivity of the displacements model

The application of a definite structure of the displacements model in the description of the kinematic state of a network requires the precise analysis of the model due to its sensitivity. The model sensitivity can be calculated by the following scalar coefficient:

$$a = \sigma_0 \sqrt{\frac{\delta}{\lambda_{min}(\mathbf{M})}} \tag{2}$$

where: a – sensitivity of a model

$\sigma_0$ – mean error of a typical observation

$\delta$ – non-centrality parameter

$\lambda_{min}$ – minimal eigenvalue matrix of the normal equation system $(\mathbf{M=A^TPA})$

The value of the above-presented coefficient (2) stands for the least absolute deformation value which may be detected by the given model. The non-centrality parameter is the basis for calculating the sensitivity of a model. This parameter was calculated according to Gil's (Gil, 1995) suggestion in this paper. The non-centrality parameter can be calculated from the correlation:

$$\delta = \frac{\Delta E}{\sigma_0^2} \tag{3}$$

where: $\Delta E$ – the increase of the square norm of the corrections' vector V determined on the basis of an observation system adjustment within the minimal limitation of the freedom degrees and the adjustment fixed on the assumption of the absolute constancy of the reference points set.

$\sigma_0$ – mean error of a typical observation

The maximum value of the non-centrality parameter reached the level of $\delta$ = 2.0 but sensitivity of a model reached the level of a=1.4 mm

## The geometry of the least square method

The adjustment issues in the linear and non-linear aspects are connected with the knowledge of the geometrical aspects of a least square method. In the case of a linear model the plane is the space estimation (fig.2) but in the case of a non-linear model the surface is the space estimation (fig.3). Minimization of the sum of the second powers corrections V(X) of a linear model aims at searching for an estimator $\hat{X}$ of parameters' vector X in order to place a given point $P(P_1, P_2, \ldots, P_n)$ located in the estimation space as a vector $\hat{L} = A\hat{X}$ as close as possible to the point L determined by an observation vector $L(l_1, l_2, \ldots, l_m)$.

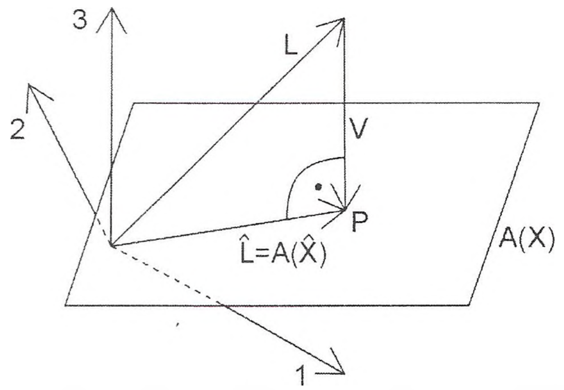


*Fig.2.* **The geometry of the least square method in the linear aspect**

In the case of a non-linear model minimization of the sum of errors second powers to observations V(X) aims at searching for such an estimator $\hat{L} = A(\hat{X})$ in the estimation space which is located closely to the point determined by an observation vector $L(l_1, l_2, ..., l_m)$ (fig.3).

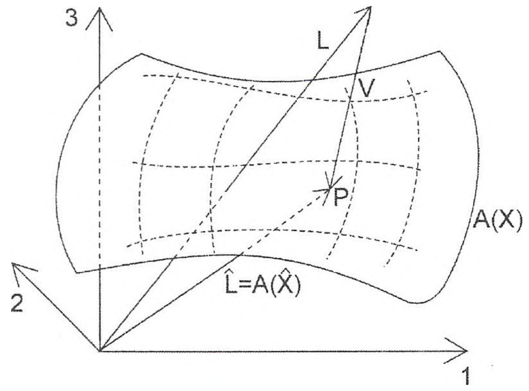


*Fig.3.* **The geometry of the least square method in the non-linear aspect**

**The non-linear feature of the displacements model**

The non-linear model can be presented in the general form as:

$$E(L) = A(X) \tag{4}$$

where: E – operator of an expected value (the average value of a random variable L)

L – observations vector

A – non-linear projection which assigns an observation vector $L \in R^m$ to the parameters' vector $X \in R^n$, provided that $m \geq n$

$R^m$ – measurements space

$R^n$ – parameters space

Most of the adjustment tasks were proceeded by the linearization of a non-linear model in the point of the known approximation $X_0$ in the form of:

$$E(\Delta L) = \partial A(X_0)\Delta X \tag{5}$$

where: $\partial A(X)$ - partial derivatives matrix of **A** projection

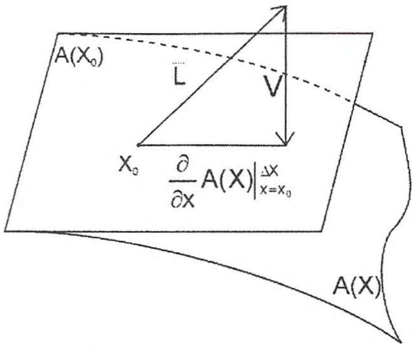


*Fig.4.* **Linearization of a non-linear model A(X)**

**A reference system identification**

The correct defining of the displacements model depends on the way the reference system is defined. Gil's method of defining a reference system (Gil, 1995) was adopted in the paper. This method consists of the two stages:

- the preliminary identification,
- the final identification.

The preliminary identification aims at determining the most possible numerous set of points which can remain the reciprocal constancy within the defined approximation. The idea of the two adjoing objects ($O^1$) and ($O^2$) represented by the two n-element sets of points $\{S^1\}$ and $\{S^2\}$ in the space $R^1$ was applied to search for an object location ($O^2$) with regard to the object ($O^1$) in order to the differences of their geometrical internal features would reach the minimal value. The optimal solution can be found on the segment between the two points or in the one of the extreme points. If the numbers of sets points $\{S^1\}$ and $\{S^2\}$ are sorted out provided that the lengths $h_i$ fulfil the condition $h_1 \le h_2 \le .... \le h_n$, hence, the function of the sum of absolute divergences reaches its minimum in the range of $h_{n/2} \le x \le h_{n/2+1}$ for the even number of points (fig.5) but for the odd number of points, the function of the sum of absolute divergences reaches its minimum in the point $x = h_{(n+1)/2}$ (fig.6).

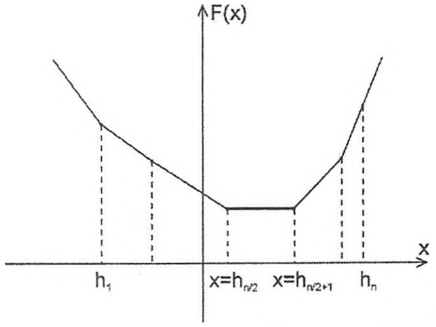


*Fig.5.* **Minimum sum of absolute divergences for the even number of points**

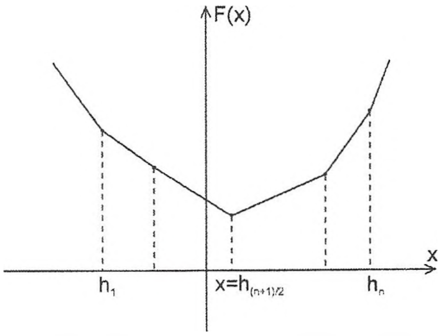


*Fig.6.* **Minimum sum of absolute divergences for the odd number of points**

Defining a reference system in the final version aims at doing research on the reaction of an observation system which is caused by the increase of points number fulfilling the condition of the absolute constancy in the process of adjustment. The differences values of geometrical features of points sets $\{S^1\}$ and $\{S^2\}$ can be sorted out according to the increasing absolute value $|w_1| \le |w_2| \le .... \le |w_n|$ on the basis of the preliminary identification. The consecutive network adjustment within the absolute constancy of its points can be done provided that the increase of the number of constant points according to the fixed order is taken into consideration. This can be done until the increase of the second power of error corrections norm reaches the critical value $\Delta E_k$:

$$\Delta E_k = -2(m^2 + \frac{m^2}{2r})\ln(1 - P^{\frac{1}{k}}) \tag{6}$$

where: P – probability value

m – mean error of a single observation defined from the adjustment within the minimal limitation of freedom degrees

r – number of supernumerary observations

k – number of points within the absolute constancy assumption

The way of identification of a reference system for the obtained measurement data was presented graphically in the fig.7.

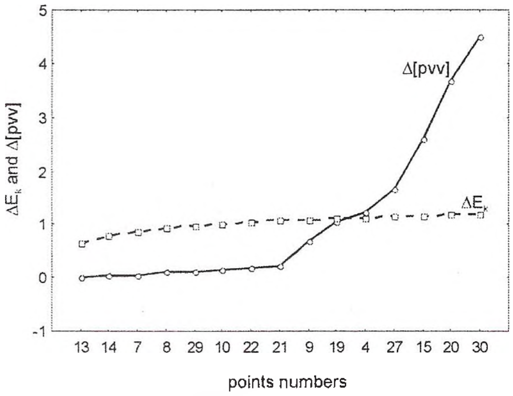


*Fig.7.* **Identification of points of a reference system on the basis of a critical value** $\Delta E_k$

Finally, the reference system was defined on the following four points 7, 8, 9, 13, which remained the reciprocal constancy in the whole period of measurement process.

**The models of kinematic networks**

The general form of a kinematic model of vertical displacements of the geodetic points network shows the following formula:

$$\Delta h = \Delta h(t, x) \tag{7}$$

where: $\Delta h$ – change of height difference

$t$ – time (real variable)

$\boldsymbol{x}$ – parameters vector

The following three kinematic models were applied in the paper:

1. $$\Delta h_1(t,\alpha) = \alpha_1 + \alpha_2 t + \alpha_3 t^2 \tag{8}$$
2. $$\Delta h_2(t,\alpha) = \alpha_1 + \alpha_2 \exp(-\alpha_3 t) \tag{9}$$
3. $$\Delta h_3(t,\alpha) = \frac{\alpha_1 t}{\alpha_2 + t} + \frac{\alpha_3}{t} \tag{10}$$

## The optimized neural network

The optimized neural network with the circular structure was used in the process of estimation of parameters of points movements in both the linear and non-linear model. This optimized neural network was applied to solve a system of equations in the form of (Osowski, 1996):

$$\mathbf{Ax} = \mathbf{l} \tag{11}$$

where: $\mathbf{A} \in R^{m*n}$ – (m>n) matrix of coefficients of equations system corrections

$\mathbf{x} \in R^{n}$ – vector of the unknowns

$\mathbf{l} \in R^{m}$ – vector of absolute terms

The task solutions aimed at defining the coefficients of parameters vector **x** which was to meet the condition (12) with the smallest error (Osowski, 1996). Hence, the objective function can be presented in the following formula:

$$\mathbf{F(x)} = (\mathbf{Ax} - \mathbf{l})^T (\mathbf{Ax} - \mathbf{l}) = \|\mathbf{Ax} - \mathbf{l}\| \rightarrow \text{minimum (for } \mathbf{x} \in R^{n}) \tag{12}$$

In order to minimize the sum of the second power of observation correction, the gradient method was also applied to solve a differential system of equations written in the matrix form:

$$\frac{\mathbf{dx}}{\mathrm{dt}} = -\mu \nabla \mathbf{F(x)} = -\mu \mathbf{A}^{\mathrm{T}} (\mathbf{Ax} - \mathbf{l}) \tag{13}$$

where: $\nabla \mathbf{F(x)} = \left[ \frac{\partial \mathbf{F}}{\partial \mathrm{x}_1}, \frac{\partial \mathbf{F}}{\partial \mathrm{x}_2}, \ldots, \frac{\partial \mathbf{F}}{\partial \mathrm{x}_n} \right]^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}} (\mathbf{Ax} - \mathbf{l})$ - the gradient vector defined by partial derivatives of the objective function with regard to individual variables

$\mu$ - neural network learning coefficient within the range of (0,1)

## The obtained results

Firstly, the values of points displacements were determined on the condition that the network was static. The calculations were done by the means of least square method within the defined reference system based on the points 7, 8, 9, 13. The calculated displacements were in the range of -5,9 - +2,2 mm (fig.8) in the course of the whole measurements periods.

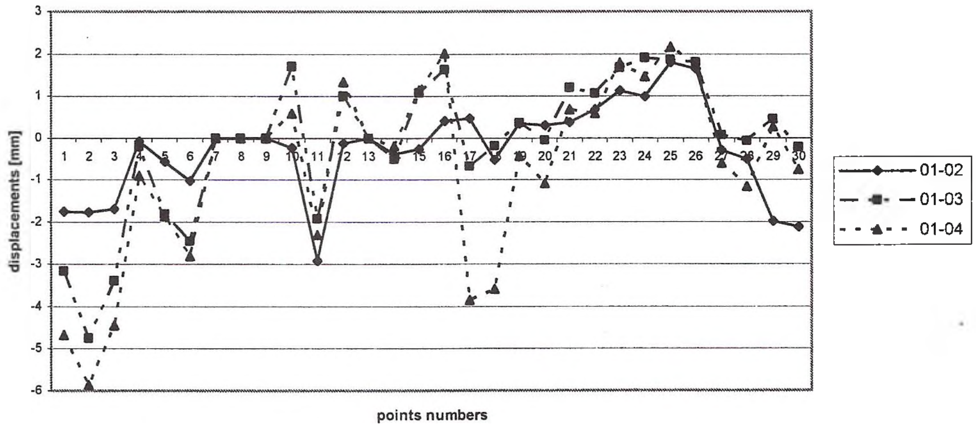


*Fig.8.* **The diagram of displacements calculated by a least square method**

The model (8) in the form of quadratic polynomial is a linear model according to the parameters vector $\mathbf{x}=(\alpha_1, \alpha_2, \alpha_3)$. The coordinates values of this parameters vector can be determined either by a classical procedure of the least square method or by a neural network. As a result of the application of the above-mentioned algorithms the deflection differences among those three models were within the range of 0,01-0,07 mm. The state of a kinematic network described by the model (8) was shown in the fig.9.

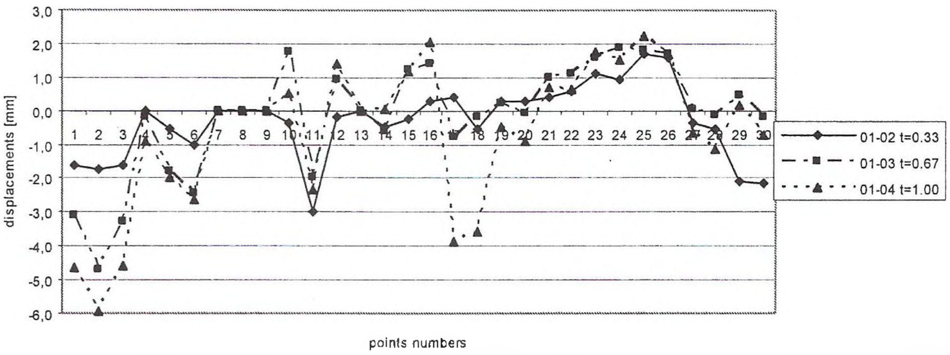


*Fig.9.* **The diagram of points displacements for the model (8) defined by a least square method and by a neural network**

The expected results were not achieved in the process of the model linearization (9) in the form of the exponential function because the mentioned model could not be transformed into a linear one. Hence, this parameters model were estimated by a hybrid method according to the following way:

- the estimation of parameters $\alpha_1$ and $\alpha_2$ of a linear model for $\alpha_3$ equalling a given constant,
- the estimation of parameters $\alpha_3$ of a non-linear model for $\alpha_1$ and $\alpha_2$ having the constant values.

The linear estimator of the vector parameter $\hat{\alpha}_1$ i $\hat{\alpha}_2$ was being searched by an iterative Jacobi's method. While the non-linear estimator $\hat{\alpha}_3$ was determined by the biggest descent method. The displacements diagram was presented in the fig.10. In the case of neural networks the problem of estimation of points movements parameters can be solved by a neural network based on the conjugation gradients method (fig. 11).

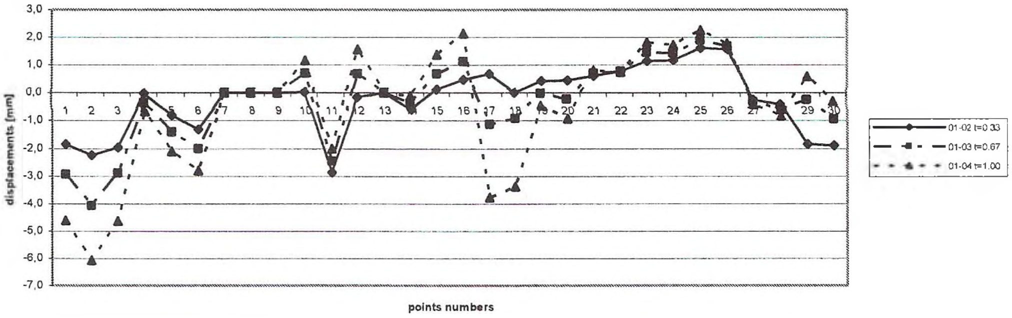


*Fig.10.* **The diagram of points displacements for the model (9) calculated by the hybrid method**

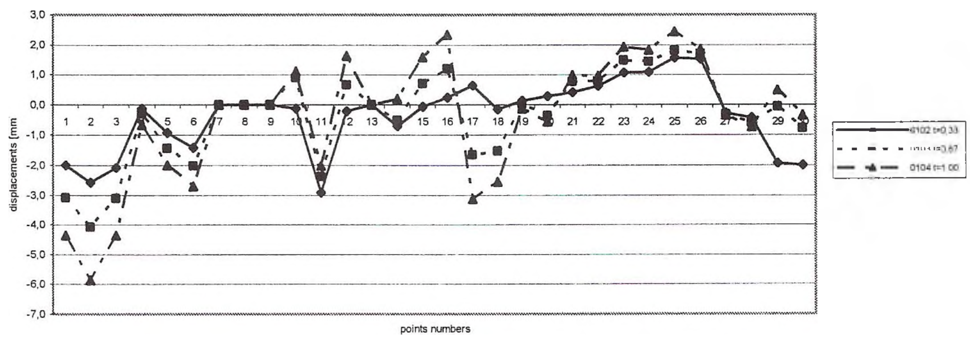


*Fig.11.* **The diagram of points displacements for the model (9) calculated by a neural network basing on the conjugation gradient method**

The difference in displacements calculated by the exponential model and the polynomial model was within the limit of 1 mm.
Michaelis-Menten's modified model (10) was compiled by a neural network with the application of a bipolar activation function with the fixed value of a slope coefficient $\beta=0,05$. The non-linear activation functions did not generally improve the process of parameters estimation. The displacements values calculated by this model were graphically presented in the fig. 12.

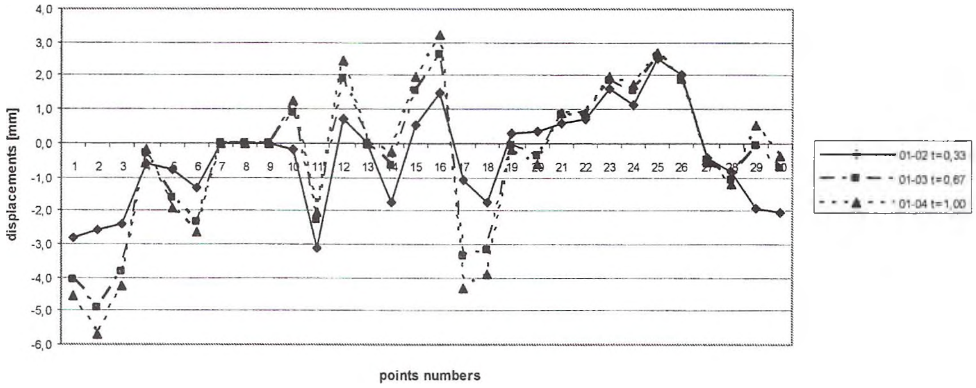


*Fig.12.* **Displacements determined on the basis of the modified Michaelis-Menten's model (10) calculated by a neural network with the application of a bipolar activation function**

The general characteristics of this model accuracy and the accuracy of a model in the form of an exponential function reached the level of a mean error of a single observation $m_0$=0,5-0,6 mm, while the coefficients differences of displacements vectors determined by those two above-mentioned models and by the polynomial were within1 ÷ 3 mm range.

## Conclusions

- The values of mean errors of a single observation $m_0$ resulted from the application of a neural network and classical algorithms were within the range of 0,4 ÷ 0,6 mm.
- The differences of displacements values determined by the above-mentioned models were within the range of 1 ÷ 3 mm provided that the whole measurements periods were taken into consideration.
- A neural network applied in estimation of kinematic models of a vertical measuring-controlling geodetic network has been an effective tool providing us with authoritative results.

## Bibliography


1. Brillouin L. „Nauka a teoria informacji", PWN, Warszawa 1969.
2. Gil J. „Badanie nieliniowego geodezyjnego modelu kinematycznego przemieszczeń (na przykładzie obciążanego podłoża gruntowego)", Wydawnictwo Wyższej Szkoły Inżynierskiej w Zielonej Górze, Zielona Góra 1995.
3. Kadaj R., „Modele, metody, i algorytmy obliczeniowe sieci kinematycznych w geodezyjnych pomiarach przemieszczeń i odkształceń obiektów", Wydawnictwo AR w Krakowie, Kraków 1998.
4. Lazzarini T. oraz zespół współautorów. „Geodezyjne pomiary przemieszczeń budowli i ich otoczenia",PPWK, Warszawa 1997.
5. Niemeier W., 1982. „Design, diagnosis and optimization of monitoring networks in engineering surveing", Proceed. Of Centennial Convention, Canadian Institute of Surveying.
6. Osada E. „Analiza, wyrównanie i modelowanie geo-danych", Wydawnictwo AR we Wrocławiu, Wrocław 1998.
7. Osowski S. „Sieci neuronowe", Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 1996.
8. Pelzer H., 1972. „Nachweis von Staumauerdeformationen unter Anwendung statistischer Verfahren", Berichte DVW Arbeitsgruppe B, Deutcher Geodätentag, Braunschweig.