APPLICATION OF MATHEMATICAL MODELS FOR THE SIMULATION OF THERMAL COMFORT CONDITIONS IN A LIVING ROOM
MATEMĀTISKO MODEĻU LIETOJUMI DZĪVOJAMĀS TELPAS TERMISKĀ KOMFORTA APSTĀKĻU MODELĒŠANAI

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Abstract. *The paper deals with the distributions of temperature and averaged turbulent airflows in living rooms in 2D and 3D approximations using Ansys/Flotran and Ansys/CFX software respectively. The distributions are calculated depending on the placement and temperature of heaters, heat transfer coefficients of the building structures and ventilation conditions. The authors analyse the influence of these factors on the air circulation and the related heat flows through building structures. The thermal balance of a room and its dependence on various external factors is also considered. As thermal comfort conditions' parameters are analysed, the airflow velocities and indoor temperatures with its gradients. It is shown that it is possible to save heating consumption, at the same time maintaining the conditions of thermal comfort in the room.*

Keywords: *Mathematical modelling, thermal comfort conditions, living rooms, temperature, airflows, heating, heat consumption, heat losses.*

Introduction

The placement of the heaters and their operating temperature essentially influence the distribution of temperature in the living rooms. Such a distribution strongly depends on the behaviour and intensity of airflows, which determine the thermal convection, controllable and uncontrollable heat fluxes through the openings of ventilation and gapes in the room's walls (e.g. crannies in window-frame), as well as heat transfer through windows, floor and ceiling. Under such conditions the heat consumption for maintaining thermal comfort increases essentially, which, in turn, leads to an increase in the heat transfer coefficient U (W/m^{2}K) of heat transfer through boundary constructions, especially through the external wall [1].

Heat transfer in a thermal boundary layer essentially increases with growth of heat flux through structure and airflow intensity near it. As a result, the thermal resistance (R_i) of boundary layer can considerably differ from the standardized value [2] for vertical indoor boundary layer R_i = 0,13 m^{2}K/W. It is possible to regulate the intensity of airflows at the vicinity of boundary surfaces and corresponding thermal resistance by choosing an appropriate placement for heating facilities and ventilation openings, as well as by adding windowsills to the windows.

In such a way it is possible to reduce the coefficient of heat transfer and the total heat leakage of the building at a fixed heat transfer coefficient of the building structure. However, in case of great heat conductivity of walls and delayed air circulation (e.g. room's nooks), heat exchange there may be reduced, what is the reason for a decrease of the surface temperature – the dew-point and water condensation can be achieved there. Adverse thermal comfort conditions in a room are notably defined by uncontrolled airflows from openings in the window-frame; this usually is the cause of increased total heat losses.

Person's feeling of comfort is fundamental impressed by velocity of airflows, absolute temperature and amplitude of the vertical temperature gradient in the room. The optimal arrangement of heaters, accordant packing of window-frames and installation of controllable venting system allows maintenance of thermal comfort in the living room with reduced heat consumption. Influence of above mentioned factors is effective and detailed analysed by use of mathematical modelling approach.

Materials and methods

The calculations have been performed for the room shown in Fig. 1 filled with air. Only one of the walls (W$_4$) has a window and a boundary with the exterior air. Heat transfer coefficient for outer wall is 0,35 W/m^{2}K and it corresponds to building requirements established in Latvia [2], but for window it is 2,5 or 6,0 W/m^{2}K depend upon modelling variant. The problem is simplified assuming that the considered room is one of many, i.e., the heat flow through the side walls is practically absent. On walls convection boundary conditions are set with according surface heat transfer coefficients [2]. The temperature in the room above and below the floor is chosen from the condition of thermal comfort, T=20°C, while the temperature behind the wall W$_3$ is chosen essentially lower: T=15°C (e.g. in the hallway). The exterior temperature behind the outer wall W$_4$ corresponds to winter conditions, T=-10°C. The surface temperature of the heater (W$_K$) is set to constant 50 or 60°C. Crannies in window-frame (W$_{O1}$) and ventilation system's opening (W$_{O2}$) as well as windowsill is geometrically created only in several modelling variants and there opening boundary condition with constant pressure is defined.
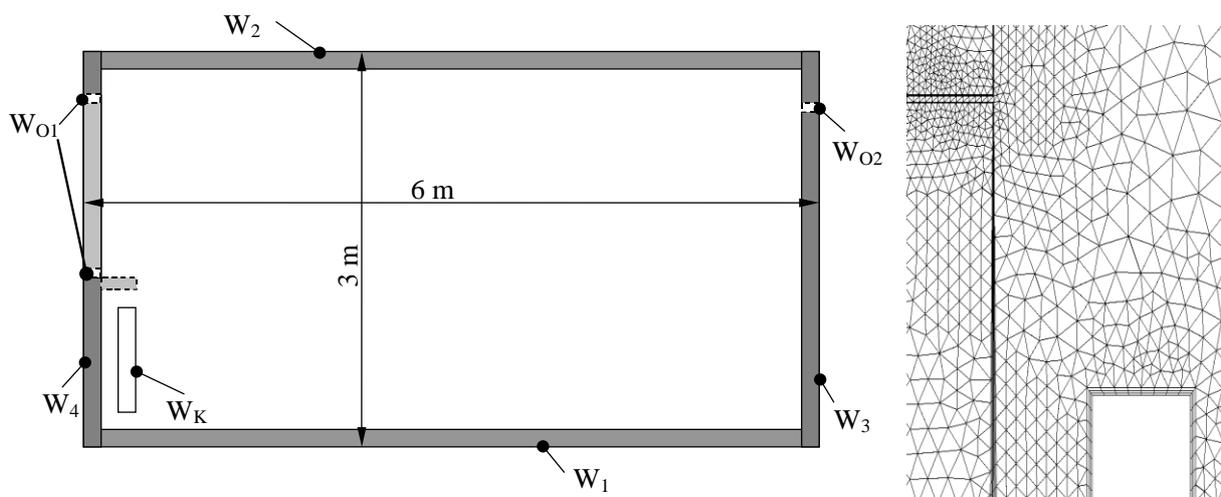

Fig. 1. **Layout of building structures in a room and illustration of discretisation**

The airflow in the room depends both on the convection created by the temperature difference and on the air exchange between openings in building structures (openings, ventilation system, etc.). To determine airflow characteristics, the following dimensionless numbers are employed [3]:

- Reynolds number, $\mathrm{Re} = v_0 L / \nu$ (v_0 is the characteristic velocity, L is the characteristic size and ν is the cinematic viscosity). Simple calculations show that in our case the Reynolds number is approx. 10^4, which corresponds to the turbulent airflow character;

- Peclet number, $\mathrm{Pe} = \nu L / a$ (a is the temperature conductivity). In current model it is approx. 10^5, which means that in the heat exchange there dominates convection;

- Prandtl number, $\mathrm{Pr} = \nu / a$. In the case of turbulent airflow the turbulent number (Pr$_T$) should be used. In our calculations Pr$_T$ = 0,85 was used.

To describe the quasi-stationary behaviour of temperature and averaged turbulent flows, traditional differential equations are employed [4]:

- Reynolds averaged momentum equation;
- continuity equation;
- equations for specific turbulence energy k and dissipation rate of this energy ε;
- energy conservation equation.

The turbulent viscosity ν_T is calculated by using the k-ε turbulence model under traditional boundary conditions [5]: $\nu_T = c_\nu k^2 / \varepsilon$, where $c_\nu = 0,09$ is an empirical constant. The temperature distribution should be determined both inside the room and in the building structures, because

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the convection type boundary conditions $\lambda \cdot \partial T/\partial n = \alpha(T-T_\infty)$ are set for the temperature at the outer boundary of these structures. The solar heat radiation through the window is ignored in order to simplify the model. In the case of a model with openings there is airflow from/to the outside thanks to pressure boundary conditions at openings. For all surfaces, except openings, non-slip boundary conditions ($v = 0$) are used.

For the numerical modelling the software package Ansys/Flotran and Ansys/CFX was applied to obtain both the stationary temperature distribution and averaged airflows in the approximation of the k–ε turbulence model. The discretisation was performed with triangular elements of varying size; boundary layers are discretised with smaller hexagonal elements. The size of finite elements is from 10 cm in the middle of the room till 1 mm in the vicinity of the heater and the openings in the walls. Therefore, the total number of elements depending on geometry and the modelling variant varies from 0,5 to 1,2 millions. An example of finite elements discretisation near the heater and wall is shown in Fig. 1.

The boundary conditions of the third type (convection from walls to the outside and to other rooms) for temperature on all surfaces of building structures and the low viscosity of air essentially worse the convergence of iteration process. The time required for calculations with a 3 GHz computer for 2D variant is 15–25 hours and for 3D variants – up to 80 hours. The difference between the heat amount from heater and the heat losses from the outer surfaces and openings of the building decreases below 10% during each simulation.

Results and discussion

Some variants of the modelling should be considered, which evidently show the impact of particular changes in the geometry and heat exchange conditions on the physical fields, characteristic values of temperature and airflows. In this way thermal comfort conditions can be analyzed.

1. Adding the windowsill and reducing of heat conduction (variants 1-1 and 1-2).

In a room without windowsill and great heat conductivity of window (U=6,0 W/m^{2}K, one-glass window analogue) the airflow from the heater is directed upwards (Fig. 3a). In this modelling variant (1-1) heat transfer coefficient for window is chosen very high to qualitatively illustrate the exchange process. As one can see, the flow of warm air from heater's surface with temperature of 60 °C is moving to the window where the intensive heat exchange takes place thanks to low temperature near the window. Characteristic thermal comfort factors and according heat losses is shown in Table 1 – the average temperature in this room with relatively high heater's temperature is about 18 °C and average velocity in whole room is up to 25 cm/s. As one can understand, the highest values of airflows are close to inner surface of the window, in the middle of the room flows are about some centimetres per second. In its turn, the greatest heat leakage is connected with outer wall and window – approx. 80% of the total losses. It is clear that in this type of rooms the thermal comfort conditions as well as economy of heating power is not reached.

One of the ways to reduce the heat losses is to increase the thermal resistance of the wall and the window, thus reducing the temperature gradient and thermal losses. Another way is to change the geometry in such a manner that the airflow was mechanically deflected from external boundary structures. For example, it can be realised by employing of windowsills.
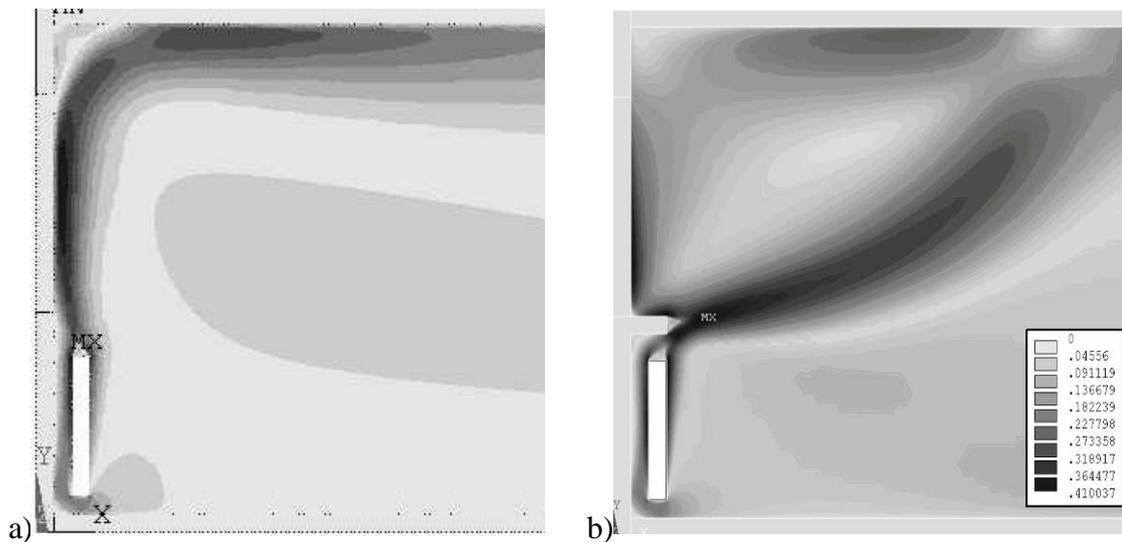
Fig. 3. **Stationary distribution of the velocity modulus in the modelled room. Images (a) and (b) correspond to variants 1-1 and 1-2 respectively**

Distribution of airflows and their behaviour in the variant with the windowsill and better isolated wall and window (1-2) is shown in Fig. 3b. The windowsill now serves as a mechanical barrier for the airflow and provides flow of the warm air in the centre of the room. A high velocity of air is observed near the surface of the window, which is attributed to the high horizontal gradient of temperature $\partial T / \partial x$ in this place, as the window is the most powerful heat conducting element in this model. Considering the motion of the air at the upper part of the convector, one can see that it is directed to the centre of the room, but relative inactive masses of warm air are present directly under the windowsill. Distribution of temperature in the area of the windowsill is shown in Fig. 4.

Table 1.

Characteristic factors and heat losses for different modelling variants

Modelling variant	Pressure difference between opposite walls ΔP [Pa]	Average temperature in the room T [°C]	Average velocity in the room v [m/s]	Average velocity v [m/s]		Heat losses through surface Q (normalized to 1 m width room) [W]				
				in the window-frame	in the ventilation opening	W_1+W_2	W_3	W_4	$W_{O1}+W_{O2}$	Total
1-1	-	18	0,25	-	-	14	27	156	-	186
1-2	-	26	0,15	-	-	8	28	118	-	154
2-1	0	22	0,1	0,13	0,10	6	6	80	34	126
2-2	5	1,5	0,32	2,3	0,47	-50	-10	19	337	295

If the heat transfer coefficient of the window is high and airflow is deflected from it, e.g., by the windowsill, then the temperature of the inner surface of the window can considerably decrease, as a result, the temperature difference between this surface and the air in the room increases thus leading to a risk of condensate formation.

As the flow of warm air deviates from the window, where the greatest thermal losses are observed, the average temperature in the room also increases. The temperature in this variant is 26°C (against 18 °C in variant 1-1). Thus, the general thermal requirement is also reduced (Table 1) – heat losses through external wall and window become only 75% from losses in variant 1-1. The mentioned temperature cannot be considered comfortable for people; however in real living rooms it is lower owing to convective thermal losses by air exchange. The maximum air velocity in the middle of the room is changed in comparison with variants without a windowsill and it not reaches 15 cm/s.
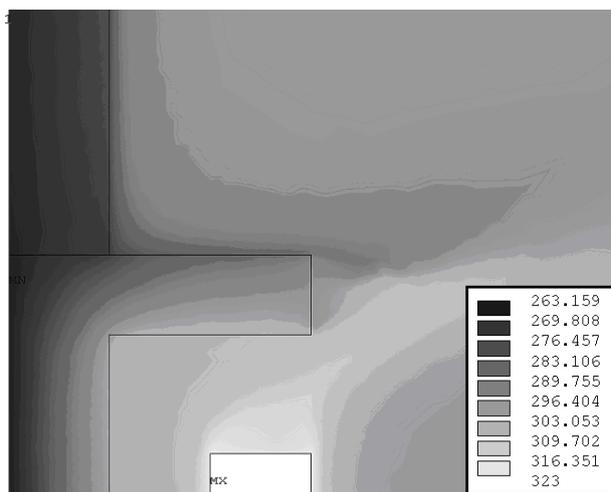

Fig. 4. **Temperature distribution near heater and windowsill in modelling variant 1-2**

Detailed analysis of modelling results for living rooms with different boundary constructions' heat transfer coefficients, varying heater surface temperature, geometry configurations and corresponding comparison of heat balances is scribed in publication [6].

2. Convective heat exchange through openings in the walls (variants 2-1 and 2-2).

In the previous variants a closed room without openings for air exchange was considered. However, in reality, rooms are not isolated from environment and the airflows compensate the consumed oxygen that is necessary for breathing. Therefore, the following designed models include openings in the boundary structures (W_{O1} and W_{O2} in Fig. 1.) which provide the air circulation. Thus, there are also additional thermal losses that have not been taken into account in the variants considered above.

To demonstrate influence of crannies in low-quality windows packing on total balance and on thermal comfort conditions, two air exchange modelling variants with different boundary conditions are considered and analysed:

- with 0 Pa pressure difference, which is the case when between the opposite walls of the room there is no pressure difference;
- with 5 Pa pressure difference. A case is considered when outdoor air pressure is higher than in the ventilation opening. This is frequent situation for real buildings.

The location of two outer openings in the model is between the window and the wall. The sizes of openings should be small to correspond to the real room situation. In the model, 5 mm openings are employed. In the opposite wall one more opening is modelled which corresponds to a 2,5 cm wide ventilation system's opening. On the lines in openings that delimit the room from the environment, the pressure values are taken 0 and 0 or 0 and 5 Pa respectively, as well as the temperature equal to T_∞=-10°C on W_{O1} and T_∞=15°C on W_{O2} assumed for the corresponding surface under third type boundary conditions. To allow flowing of air through openings, non-slip conditions are not used on these lines.

In contradistinction to previous models with one-glass window, in variants 2-1 and 2-2 window with two glasses is used (U=2,5 W/m^{2}K) and no window-sill is created. As a result heat conduction losses trough it are reduced, hence total heat losses to outside decreases too (Table 1). Although cold outer air flows through crannies in window-frame exists even in case with 0 Pa pressure difference, thermal comfort conditions (temperature about 22°C and maximal airflow intensity in the openings is less than 15 cm/s) in a room are reached with heater's constant surface temperature 50°C. It is very essential that in this case, when conditionally calm weather outside room is set up, the fresh air flow necessary for breathing is provided; at the same time openings do not generate extra heat losses by convection.

However, situation radically changes in model with ΔP=5 Pa (variant 2-2). In this case maximal airflow velocity in openings reaches 2,5 m/s and they are also intensive near heater. At the same heater's temperature (50°C) heat convection from its surface is increased more than 2 times thanks to high velocities of airflows in the room and significant low average temperature in it – only 1,5°C at outside air temperature -10°C (Table 1). Heat conduction losses through walls in this variant are only 7% and the room is heated by neighbouring rooms with constant temperature 20°C. Except for grown heat consumption, comfort conditions in this room become absolutely unsuitable for human living.

Moreover, character of air circulation between the outer wall and the heater radically changes – in case with insignificant air inflow (variant 2-1) there dominate upward airflows. However in variant 2-2 downward cold air flows from crannies in window-frame are determinant (Fig. 5). Such character of airflow substantially amplify vertical temperature gradient in the room in comparison with variant 2-1 – the floor is additionally cooled, but relatively warm air layer is remained only near the ceiling (Fig. 6); in this case thermal comfort conditions in the room are explicit uncongenial. Above mentioned analysis shows how conditions in the room are affected by uncontrolled air inflows trough crannies in the window-frame, e.g. temperature and airflows inside the room can be radically changed by the wind intensity and direction outdoors.
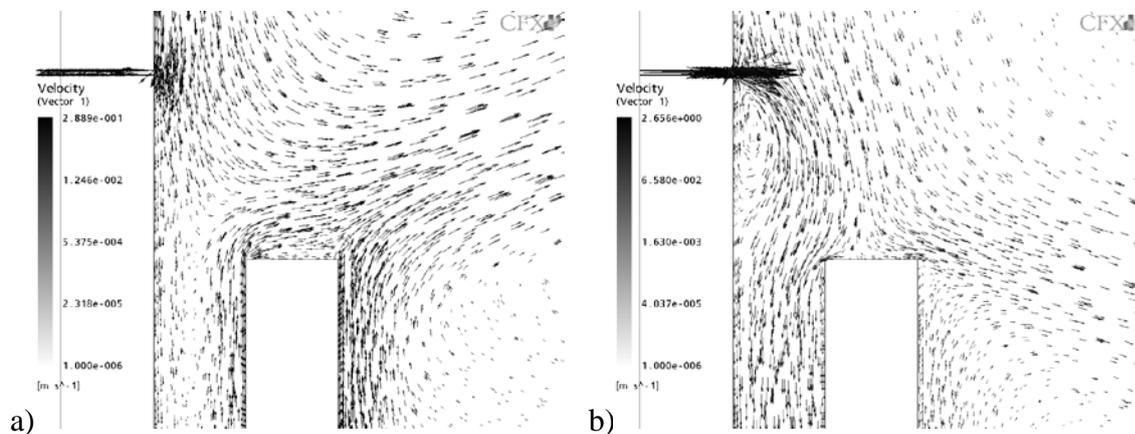

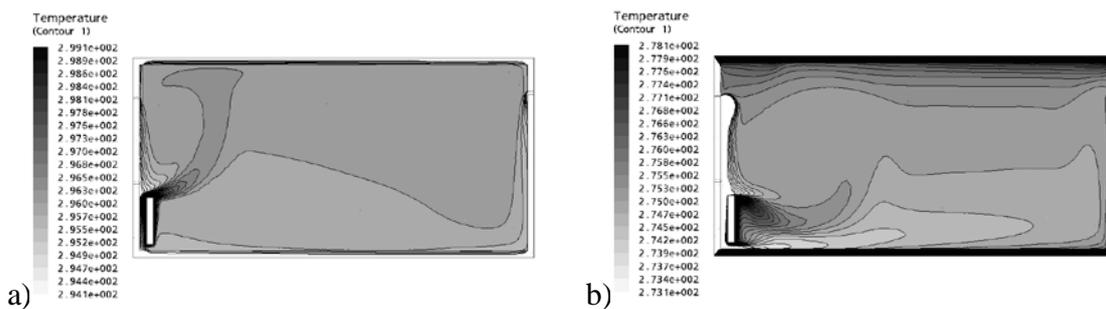*Fig. 5.* **Vectors of airflow near the outer wall and heater in variants 2-1 (a) and 2-2 (b)**

Fig. 6. **Temperature distribution in a room in variants 2-1 (a) and 2-2 (b)**

Summary of characteristic average temperatures in the whole room (include air in the openings) and corresponding heat requirements for all modelling variants are shown in Fig. 7. As one can see, heat requirement in variant 1-2 is reduced at the same time average temperature in the room grows compared to variant 1-1. But 5 Pa pressure difference in variant 2-2 critical levels down the temperature and increases necessary heat amount up to 2 times in comparison with variant 2-1.

3. 3D modelling

3D modelling, which is more numerical capacious for calculus than 2D modelling, allows to realize more exact model and accurately disclose and analyse the heat exchange processes in a real room. In a spatial modelling it is possible to visualize some effect that is not possible in 2D models – e.g. air inflow distribution through crannies in the window-frame is noticeably spatial. The outside pressure value is set to 1 Pa in this modelling variant (3-1) and on ventilation it is set to 0 Pa, thereby total pressure difference ΔP is

Fig. 7. **Characteristic average temperature and velocity in the middle of a room**

1Pa, what agrees with real natural conditions with slow breeze outdoors. Because of outdoors overpressure the air flows in through 5mm tight opening and the its velocity reaches 1,5 m/s. The velocity decreases farther from wall and is less than 10 cm/s in the middle of the room – isosurface with absolute velocity value of 0,55 m/s is visualized in Fig. 8a.

The window-sill greatly blocks hot airflow from heater, thereby air temperature near window's surface with heat transfer coefficient U=2,5 W/m^{2}K is noticeably lower than surface temperature of wall with U=0,35 W/m^{2}K. The temperature near air inflow area is considerably lower – isosurface with temperature of 7°C demonstrates that lowest temperatures exist in lower corners of window where greatest air velocities (Fig. 8b) are present. Relatively dense air after inflowing through opening continues to move down along edges of the window-sill.
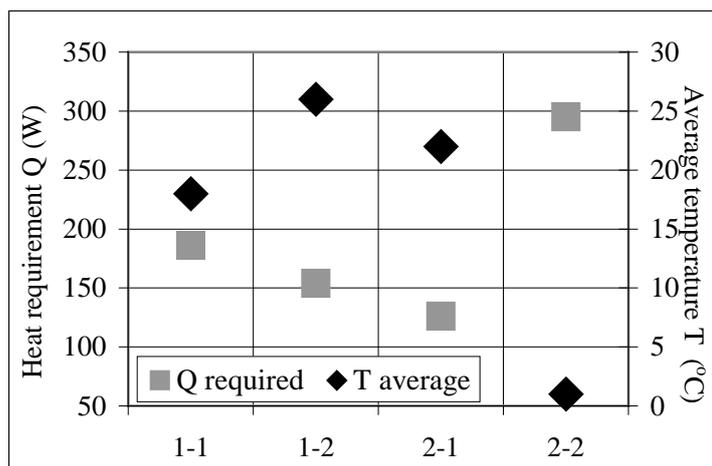

a)   b)

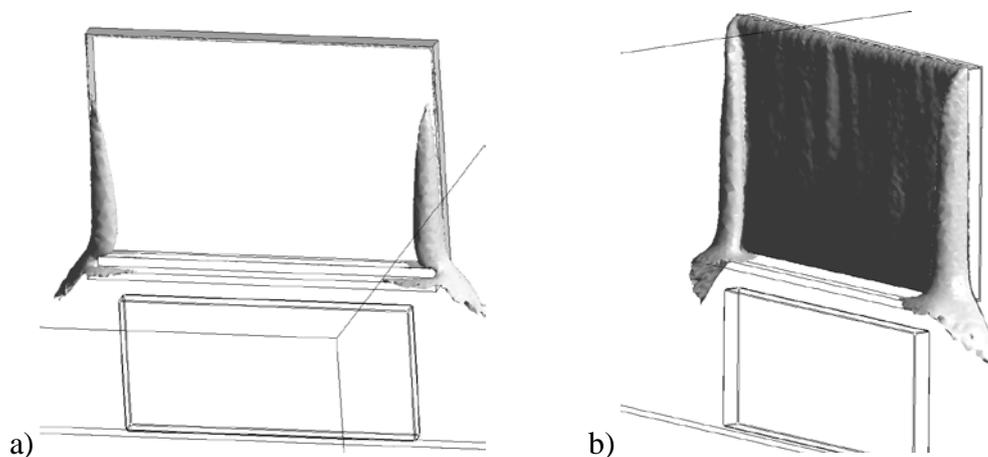*Fig. 8.* **Isosurfaces of v=0,55 m/s (a) and T=7°C (b) for modelling variant 3-1**

The air exchange intensity near the upper part of window is noticeably lower – this is due to impact of two processes. One of them is aforesaid pressure difference that promotes outdoor's air inflow. Other process is determined by convective airflows along window surfaces:

- indoors warm air becomes cold near window with great heat conductivity and moves down along its surface, in that way it generates traction for cold outdoors air inflow trough crannies in the lower part of window-frame;
- at the same time outdoors air becomes warm near outer window surface and moves up, what helps to create additional traction for air outflow in the upper part of the window.

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If there is not an extra pressure difference then typical vertical temperature distribution develops in a window, as well as air outflow appears in the upper part of openings and inflow appear in the lowest section. Aforementioned situation may change if there is no window-sill present which helps to prevent direct airflow from the heater.

Conclusions

The 2D and 3D calculations of airflows and temperature distribution in a living room show the influence of rearrangement of structural elements of a room on the character of airflow velocities and its directions in this room. As shown, it also influences the temperature field distribution, because of the heat exchange variation conditions near the building structures.

Openings in the room's walls lead to convective heat losses, which through pressure difference radically change the physical fields' distribution in the room. In this case additional convective heat losses arise, which are usually greater than those by heat conduction. The distribution of heat losses for rooms is confirmed by measurements of such parameters as airflow velocities and temperatures in different places of the room.

The above-mentioned modelling method enables to choose (in the design stage) appropriate structural elements adequately - either of the building or its separated rooms – that have the desired values of thermo-technical parameters. The model of a separated room shows the influence of various kinds of factors on the resulting distributions of thermo-physical parameters in the room that are directly related to the conditions of thermal comfort. Since the model allows, at the same time, the heat consumption to be reduced, its use may help to reduce energy production and the related pollution.

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