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Design of Permanent Magnet Linear Synchronous Motor driving 2D Table for Laser Marking

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Abstract. In this paper, the results from design of a permanent magnet linear synchronous motor are published. The motor will drive a table for laser marking of various small details. This table will be part of the equipment of Laser Technologies Laboratory of the Research Centre for Physical Processes and Laser Technologies at Rezekne Academy of Technologies. In the design process, a comprehensive approach was used in which the theory of electric and magnetic circuits was combined with Finite Element Method (FEM). The paper contains the main equations of the design methodology. They were used for the preliminary calculation of motor magnetic system and main electromagnetic parameters via script program. For the specifying of these quantities and motor optimization, the results from modeling and analyzing of the linear motor magnetic field by the FEM were used.

Keywords: Laser Marking, Permanent Magnet Linear Synchronous Motors, Design, Finite Element Analysis.

I. INTRODUCTION

According to the World and European standards the products of mechanical engineering there should be marking such as - matrix code, bar-code, logo of firm, serial numbers, basic technical features etc.

Good quality laser marking is of particular importance to both manufacturers and customers. It is needed by manufacturers so that they could be able to monitor all stages of the production cycle the other hand, it must provide the customer the necessary information about the parameters and characteristics of the products [1, 2, 3].

The laser marking propulsion system must have excellent dynamic performance - speed, acceleration in order to provide high accuracy positioning. Belt, gears and helical gears powered by servomotors with rotation motion does not give good results. A very good solution for industrial laser propulsion systems is the use of Permanent Magnet Linear Synchronous Motors (PMLSMs).

These motors enable direct conversion of electrical energy into mechanical energy of linear motion. This leads to better precision, higher acceleration and higher speed of the moving part.

In laser systems for industrial applications there are two main constructive solutions: a movable laser head and a stationary table with the work piece or stationary laser head and movable table with a detail attached thereon.

The laser system developed by Latvia Academy of Technologies will be used for marking of details with small weight and sizes. Therefore, the second constructive solution is considered in this work. In this regard, the article presents the results of the design of linear servo motor which drives a two-coordinate table to which the marked detail is affixed.

The designed linear synchronous motor shall have very good dynamics and positioning accuracy. The following dynamic parameters are required: velocity – 3 m/s; acceleration – 60 m/s^2; thrust – 180 N; positioning accuracy -100 μm.

As a drive system of this 2D table, an iron core flat type PMLSM is chosen.

PMLSMs are a complex electromechanical system characterized by strong non-linearity in terms of their electromagnetic field. Their design, as well as other electrical machines, is related to solving complex problems caused exactly this non-linearity. An appropriate numerical method (FEM) can be very useful, in the design stage, in order to obtain more information about these non linearities.

In the present work we describe the design and simulations results for the linear synchronous servomotor illustrated and intended for the needs of the laser marking system. The simulations were based on the results of the modeling and analysis of motor magnetic field with FEM. Based on these results, the electromagnetic forces acting on the moving part have been calculated as they are key parameters in motor design.

In the design process a comprehensive approach was used in which the theory of electric and magnetic circuits was combined with FEM. This combination gives excellent results. For this purpose, a special

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script program was used, through which, using the theory of electric and magnetic circuits, initial size of the magnetic system was calculated; then physical experiment was simulated based on iteration procedure using FEM. In this simulation, linear motor thrust was calculated using the results from the magnetic field analysis. Its value is used as a criterium for ending the iteration procedure when it reaches the desired value of this force

FEM was also used in the motor final design via random optimization method.

II. DESIGN OF PMLSM

The design of PMLSM was made applying a comprehensive approach comprising three main stages:

1. Initial analytical calculation of the magnetic system, the fundamental electromagnetic loads, and motor main sizes and parameters, using methodology based on the theory of electric and magnetic circuits.

2. Analysis of the motor magnetic field with the resulting geometry and physical experiment simulation with FEM, aimed at clarifying the values of item 1 in one iteration procedure with main criteria the value of the linear motor thrust .

3. Optimization of the resulting motor geometry and determining the final values of motor geometric dimensions and electromagnetic parameters.

A. Selection of construction and control of the linear motor under design

The designed linear motor was selected to be with flat type ferromagnetic core and permanent magnet excitation. Its structure is shown in Fig.1.

The movable part 1 is supported by two linear guides 2, 3 mounted on the stationary motor part 4. The mover includes the core 5 made from silicon steel. On the motor back iron NdFeB permanent magnets 6 (PM-s) are mounted, with alternating polarity N-S-N-S....

A very important issue in linear motors design is the right choice of the number of poles/number of core teeth ratio. The number of poles is directly related to the period of the magnetic detent force. This force disturbs the motor control because generates motor thrust fluctuations and vibrations in motor operation. In order to avoid any adverse effects of the magnetic detent force presence, it was decided that the motor will have 10 poles, i.e. $2p = 10$.

The ultimate target in the design of modern electrical machines aspiration is that they be energy efficient. In order to reduce copper losses of the designed linear motor, two-layer concentric winding was used, wherein each tooth of the mover core has a coil. The individual coils are connected in such a way as to form a three phase winding 7, able to create a "running" magnetic field.

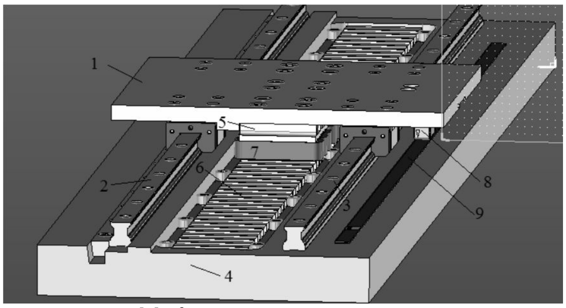

Fig1. 3D CAD model of PMLSM

In the concentric windings the number of slots per pole and per phase q is less than one. To use concentric windings it is very important to choose the correct number of armature slots that correspond to the set number of poles. From [4] a core of the movable part that has $z = 12$ slots was selected.

The vector control of the linear motor was selected. It was implemented with the help of modern digital controller JONI [5], powered by a DC voltage source. It generates a symmetrical three-phase voltage system, supplied to the motor mover via wire connection.

To monitor the position of the motor movable part and the proper work of the digital controller, the linear motor is provided with a magnetic linear encoder [6], which consists of a reading head 8 and a magnetic tape 9.

B. Design Methodology

The methodology applied is based on analytical equations from the theory of electrical and magnetic circuits in integral form and the theoretical mechanics, and from the theory of electromagnetic field in differential form as well.

The starting point for determination of the required motor thrust is Newton's law

$$F = m\,a + \mu\,g\,m\,. \tag{1}$$

where

m is the mover mass and the mass of the pay load;

μ is the friction coefficient of the used linear guide;

a, g is the acceleration of the mover and the earth acceleration.

The calculation of the acceleration a depends on the selected profile of the speed-time curve of the linear motor movable part - triangle or trapezium. It should be borne in mind that the optimal profile is triangular.

Linear synchronous servomotors permit overloading of 3÷4 times. Usually, they work with a small Duty Cycle DC, %. Therefore, they can be designed for a smaller thrust, equal of the continuous force, calculated by the following equation

$$F_{\text{continuous}} = \frac{F_{duty}}{\sqrt{\frac{DC}{100}}} \, . \qquad (2)$$

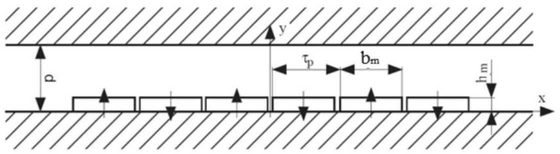

Fig.2. PM magnetic way with one infinite opposite yoke

Output parameter for the calculation of the linear motor magnetic system shown in Fig. 2 is the value of magnetic flux density in the air gap. In order to derive an analytical equation for its calculation, the PM magnetic way must be modeled. This is done by replacing each PM with two current densities of different signs. After using the image theory both B_x and B_y components of the magnetic flux density are obtained.

$$B_{x\,N_{pm}}(x,y) = \sum_{i=0}^{N_{pm}} (-1)^{i+1} B_{x\,1_{pm}\,2p}(x,y,xi,0) \;; \qquad (3)$$

$$B_{y\,N_{pm}}(x,y) = \sum_{i=0}^{N_{pm}} (-1)^{i+1} B_{y\,1_{pm}\,2p}(x,y,xi,0) \;, \qquad (4)$$

where

N_{pm} is the number of PM in magnetic way;

$B_{x\,N\,pm}$, $B_{y\,N\,pm}$ are the components of magnetic flux density.

The x - coordinate of the PM center is

$$x_i = -\frac{N_{pm}-1}{2}\tau_p + i\,\tau_p \, . \qquad (5)$$

The preliminary value of magnetic flux in the air gap is

$$\Phi_\delta = \tau_p \, \ell_\delta \, B_\delta \, , \qquad (6)$$

where l_δ is the width of the mover core and τ_p is the pole pitch.

On the base of magnetic flux value in the air gap, the mover yoke height h_a, and the number of phase turns N_{ph} was calculated

$$h_a = \frac{\Phi_\delta}{2 k_{Fe} B_a \ell_\delta} \, ; \qquad (7)$$

$$N_{ph} = \frac{k_E \, U_{ph}}{\pi\sqrt{2}\;f\,k_w\,\Phi_\delta} \, , \qquad (8)$$

where

B_a is the value of magnetic flux density in the mover yoke;

U_{ph} is the phase voltage, supplied from the inverter;

k_w is the armature winding factor.

The preliminary value of coefficient of induced voltage k_E is

$$k_e = \frac{E_1}{U} \, . \qquad (9)$$

where E_1 is the rms value of motor e.m.f.

The equivalent frequency of the supply voltage received from the motor controller, determining the linear speed of movement ν, is

$$f = \frac{\nu}{2\tau_p} \, . \qquad (10)$$

The height of the back iron is calculated by the next equation [4]

$$h_{backiron} = \frac{B_r \, b_m \, h_m}{B_{back}\,(\mu_r (d - h_m) + h_m)} \, . \qquad (11)$$

The MMF for the full magnetic circuit is

$$F_{sum} = F_a + F_Z + F_\delta + F_{backiron} \, , \qquad (12)$$

where

F_δ is the air gap MMF;

F_a - the mover yoke MMF;

F_z - the MMF for the mover core teeth;

$F_{backiron}$ - the backiron MMF.

The coefficient of saturation of the magnetic circuit is

$$k_\mu = \frac{F_{sum}}{F_\delta} \, , \qquad (13)$$

In the PMLSM design the fact that it will be supplied and controlled by a digital servo controller, realizing modern vector control was taken into account. This controller provides maximum thrust when the current $I \approx I_{aq}$ is in phase with the e.m.f induced in the winding, and, moreover, its component I_d along the d- axis is zero. Accordingly, the vector diagram was drawn as shown in Fig.3. In Fig.4 the equivalent circuit of one motor phase is shown. It is used for the calculation of basic electrical parameters of the linear motor.

The armature winding resistance for one phase is

$$R_1 = \frac{\rho_{Cu} \, l_{mean} \, N_{ph}}{S_{Cu}} \, , \qquad (14)$$

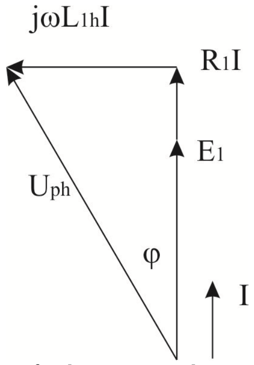

Fig.3. Phasor diagram for the maximum thrust

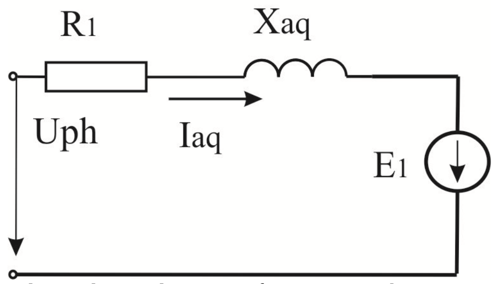

Fig.4. Electrical equivalent circuit for one motor phase

where

ρ_{Cu} is the resistivity of copper;

l_{mean} - the average length of a turn;

S_{Cu} - the conductor cross section.

The self inductance for one phase is

$$L_{1h} = \frac{4\mu_0}{\pi^2}\frac{l_m \tau_p}{p\delta'}(N_{ph}k_w)^2, \quad (15)$$

where

$\delta' = k_\delta \delta$ is the equivalent air gap;

k_δ - the Carter's coefficient;

l_m - the axial length of PM.

The value of induced phase voltage is [5]

$$E_1 = 4\ f\, k_\mu\, \Phi_\delta k_w, \quad (16)$$

The phase voltage is [7]

$$U_{ph} = \sqrt{(E_1 + R_1 I)^2 + (2\pi\ f\ L_{ph})^2}\ . \quad (17)$$

The final value of coefficient of induced voltage

$$k_E = \frac{E_1}{U_{ph}}. \quad (18)$$

With the so calculated value of this coefficient, the number of turns for one phase is recalculated and iteration procedure is organized until reaching its set value.

By the phasor diagram of Fig.3 the power factor is calculated

$$\cos\varphi = \frac{E_1 + R_1 I}{U}. \quad (19)$$

The consumed active power from the PMLSM is

$$P_1 = mU\,I\cos\varphi, \quad (20)$$

where m is the number of phases.

Iron losses are the sum of hysteresis losses and eddy current losses

$$P_{Fe} = P_h + P_e = \left[k_{Feh}c_h\left(\frac{f}{50}\right)B^2 + k_{Fee}c_e\left(\frac{f}{50}\right)^2 B^2\right]m_{Fe} \quad (21)$$

where the coefficients of hysteresis losses are k_{Feh} =1…2 и c_h = 2…5 Ws/T^{2}kg, and the coefficients of eddy current losses are k_{Fee} =2…3 and c_e=0,5…23 Ws^2/T^{2}kg .

The values of all these coefficients are generally given by the manufacturers or they can be deduced from experimental measurements.

The copper losses for the motor winding are

$$P_{Cu} = mI^2 R_1, \quad (22)$$

where m=3 is the number of motor phases.

The linear motor mechanical losses are primarily the friction losses in the linear guide and are given by the following equation:

$$P_{meh} = (m_{mover}\, g + F_{att})\,\mu\, v, \quad (23)$$

In their calculation the great force of attraction to PM should be taken into account. For determining this force, the equation the Maxwell was used [4]

$$F_{att} = \frac{2p\tau_p w_m B_{m1}^2}{4\mu_0}. \quad (24)$$

The resistivity of the NdFeB magnets is rather low ($\rho_{PM} \cong 100.10^{-6}\ \Omega/m$). Therefore, the eddy current losses in conductive PM due to magnetic fields produced by the mover slots and the coil MMF cannot be neglected. These losses are generated by the high harmonics of the magnetic flux.

Due to this reason, special attention is paid to the calculation of eddy current losses in PM. This is done basing on the equations of the electromagnetic field in integral form in result of which the following equation is obtained

$$P_{pm} = \frac{\left(\frac{2l_m\omega_{pm}B_m\,\tau\,\pi}{2\sqrt{2}}\sin\left(\frac{b_m\pi}{\tau}\right)\right)^2}{R_{pm}}, \quad (25)$$

where R_{pm} is PM equivalent resistance for eddy currents [7]

$$R_{pm} = \rho_{pm}\frac{2l_m}{h_m b_m/2}. \quad (26)$$

For the motor efficiency one can write

$$\eta = \frac{P_1 - P_{Cu} - P_{Fe} - P_{meh} - P_{pm}}{P_1}. \quad (27)$$

In the preliminary design of PMLSM, the approach to analytical calculation of the motor thrust F_x is of particular importance. Various reference sources [8], [6] offer different equations. Practical experience in the design of linear synchronous motors of flat type with ferromagnetic core and excitation of PM proves that the following equation renders very good results [4]

$$F = l_m p \tau_p H_{tm} B_{nm}. \quad (28)$$

It is received on the basis of Maxwell Stress Tensor T_m, which gives the power per unit area, created by the magnetic field acting on the surface S

$$\vec{F} = \oint\!\!\oint_S \vec{T}_m d\,\vec{s}\,. \quad (29)$$

The normal and tangential components of the tensor T_n and T_t, acting on the surface S with some simplifications can be presented in the following way

$$T_n = \frac{1}{2}\mu(H_n^2 - H_t^2); \\ T_t = \mu H_n H_t\,. \quad (30)$$

In general, the the air gap magnetic field is described by the non-sinusoidal functions because of its unequal magnetic conductivity due to the presence of teeth and slots. As decomposition of functions in Fourier's series is used, sinusoidal distribution of the scalar magnetic potential in the air gap can be assumed. Under this assumption, the tangential component of the magnetic field intensity can be presented as

$$H_{tm,\nu} = \frac{\pi V_{m,\nu}}{\tau_p}, \quad (31)$$

where ν is the harmonic number.

Scalar magnetic potential created by the three-phase motor winding is [4]

$$V_{m,\nu} = \frac{m\sqrt{2}\,N_{ph}\,k_w\,I^{\nu}}{\nu \pi\, p}. \quad (32)$$

Equation (31) gives linear current density at $\nu = 1$, i.e. for the main harmonic.

C. FEM Simulations

At the design stage of the PMLSM, an important question arises - "Do the calculated main motor sizes, its electromagnetic loads and parameters satisfy the technical assignment – i.e. whether it achieved the required thrust (being the most important parameter) acting on the mover?".

To adequately answer of this question, there are two ways:

1. Produce a prototype and experimentally measure its thrust and other parameters of interest - speed, acceleration, efficiency, power factor, etc.
2. Simulate a physical experiment by analyzing the motor magnetic field and, on the basis of the results obtained, calculate adequately this force and these parameters.

The first method is laborious and associated with considerable expenditure of time, labor and money. In the presence of powerful computing equipment and appropriate software nowadays the second method is used, which is commonly called "simulation" of the designed electromagnetic objects. It is much faster and allows for saving significant labor and financial costs. For the purpose of the simulations, the stationary magnetic field of the motor was analyzed. This field is excited by equivalent DC currents equal to the value of the actual peaks of the phase currents. Upon their setting for the three phases, it must be kept in mind that the winding is star-connected, the three phases are shifted 120^0, and currents satisfy the first law of Kirchhoff

$$I_A + I_B + I_C = 0\,. \quad (33)$$

The stationary magnetic field is described by its equations in differential form. After the appropriate transformations, the second degree differential equation for the vector potential of the magnetic field is obtained

$$-\frac{1}{\mu}\vec{\nabla}^2 \vec{A} = \vec{J}\,. \quad (34)$$

This equation is solved using FEM, the program FEMM [9] in particular. In its environment, the numerical model for analyzing the linear motor magnetic field is constructed.

To ensure the uniqueness of the solution via FEM, it is necessary to define the relevant boundary conditions, shown in Fig.5.

PMLSM can formally be derived from their rotating counterparts if cut through the axis and extended in one plane. This enables the usage of periodic boundary conditions for the analysis of the magnetic field of linear motors in the FEA.

These boundary conditions make it possible to connect the two end surfaces of motor movable and fixed part - the front and rear surfaces along the direction of movement. Their traces in the XY plane are the contours K2 and K3 in Fig.5. This method allows for the simulation of linear synchronous motors то be reduced to a simulation of rotary motors.

This is true, however, in the case when simulation of the movable part movement is not required.

For this purpose, the vertical boundary contours K_1 and K_3 in Fig.5 must be absolutely identical in structure, i.e. the lengths of the lines shall be equal. Along the other two border contours K_2 and K_4,

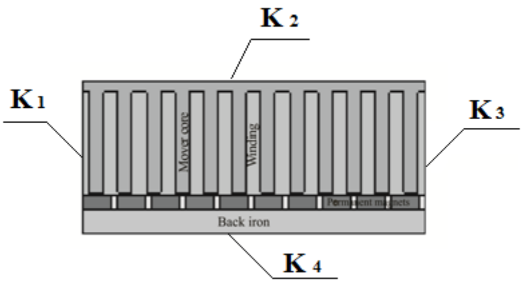

Fig.5 Counturs for boundary conditions

Dirichlet boundary conditions for the magnetic vector potential are set.

As a result of the magnetic field analysis the values of the magnetic vector potential, the magnetic flux density, and intensity of the magnetic field of the discretized region are obtained. The distribution of the magnetic flux density is shown in Fig.6.

The main output value for the electromagnetic system, called PMLSM, is the propulsion force it generates. For its calculation the FEA results are used. It is equal to the x-component of the force, calculated via Maxwell Stress Tensor with integration by the volume of the moving part.

For that purpose, iteration cycle is organized over its value by changing the phase currents of the motor. The needed design value of propulsion force is the main criterion for successful fulfillment of the iteration procedure.

Further, in accordance with the phase currents values reached, the final sizes of the mover core and the main motor electromagnetic parameters are specified.

For the design purpose, a scripted program is written running in the medium of the free mathematical software Octave [10].

Its flow chart is shown in Fig. 7.

In the first part of the program, calculation of preliminary sizes of the linear motor magnetic system and its main electromagnetic parameters is performed. In the second part, using the iteration procedure, the value of the propulsion force is calculated basing on the results of the FEA of the motor magnetic field.

The results from the program execution are summarized in Table I.

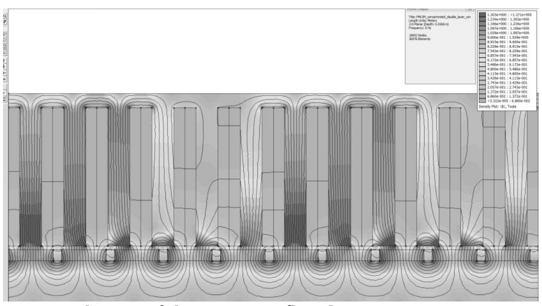

Fig.6 Distribution of the magnetic flux density

A. Optimization of PMLSM

The purpose of the optimization of the designed PMLSM is to optimize the geometry received from the computer aided design, especially the geometry of the moving part so as to reduce its size and weight in order to improve the motor dynamics.

In the present work, the problem of optimal design of PMLSM is reduced to using the method of random optimization (RO) [11]. RO is a family of numerical optimization methods that do not require the gradient of the problem to be optimized and RO can hence be used on functions that are not continuous or differentiable. Such optimization methods are also known as direct-search, derivative-free, or black-box methods.

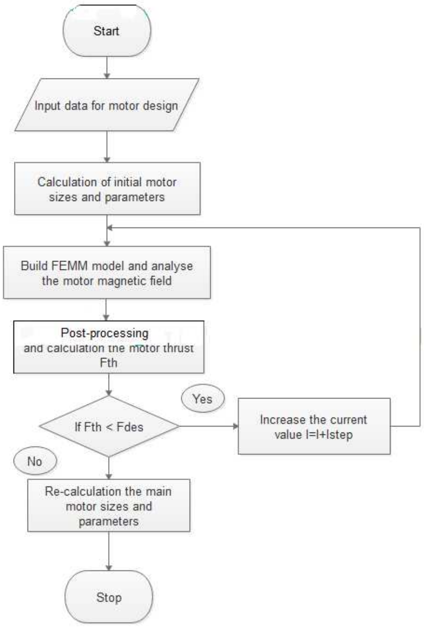

Fig.7 Flowchart of program

Table I
Rersults from the design of PMLSM

N	Quantity	Value
1	Number of phases	3
2	Number of poles	10
3	Slots number	12
4	Rated phase voltage [V]	50
5	Rated phase current [A]	9
6	Eficiency	0.85
7	Power factor	09
8	Air gap [mm]	1
9	Pole pitch [mm]	15
10	Tooth pitch [mm]	12.5
12	Hight of back iron [mm]	8.5
13	Hight of mover yoke [mm]	4
14	PM hight [mm]	6.35
15	PM length [mm]	76
16	PM width [mm]	12.7
17	Speed [m/s]	3
18	Mover mass [kg]	2

Table II
Optimal values of the optimization variables

N	Variable	Optimal value
1	The hight of backiron [mm]	7.9
2	The hight of mover yoke [mm]	4.23
3	PM hight [mm]	6.11
4	PM length [mm]	7.59
5	The motor active volume [m^3]	0.000506
6	Thrust [N]	180

The objective function which must be minimized is the motor active volume, calculated as follows

$$V_a = 2\, p\, \tau_p\, l_m\, (h_{back} + h_m + \delta + h_k + h_a)\,. \quad (36)$$

The vector of optimization variables is

$$[\vec{x}] = [h_{back}\; h_m\; h_a\; l_m\;]^T\,. \quad (37)$$

Thus optimal design of PMLSM is reduced to solving of the following problem:

Finding the minimum of the function

$$f(\vec{x}) = V\,, \quad \vec{x} \in E^4\,, \quad (38)$$

with the following optimization inequality constraints:

$$\left.\begin{aligned} h_{back} &\le 10\ mm; \\ h_a &\le 9.5\ mm; \\ h_m &\le 6.35\ mm; \\ l_m &\le 76.2\ mm. \end{aligned}\right\}. \quad (39)$$

These are constraints-inequalities of the kind $g(\vec{x}) \le g_{\lim}$ imposed by technological and structural reasons, mostly related to the motor production. Optimization procedure continues until the extremum of the objective function is reached

$$y(\overset{*}{\vec{x}}) = y(\vec{x})_{extr}\,, \quad (40)$$

defined by the vector with the optimal values of the variables.

The optimal values of the optimization variables are shown in Table II.

III. DESIGN OF 2D LASER TABLE

The designed PMLSM will be used to drive a two-coordinate table, on which the workpiece is attached, subject to laser marking. Its 3D CAD model is shown in Fig.8.

Table III
Technical data of the table for laser marking

N	Parameter	Value
1	XY Travel distances [mm]	300
2	Maximum linear speed [m/s]	1
3	Maximum pay load [kg]	1
4	Position accuracy [μm]	± 20

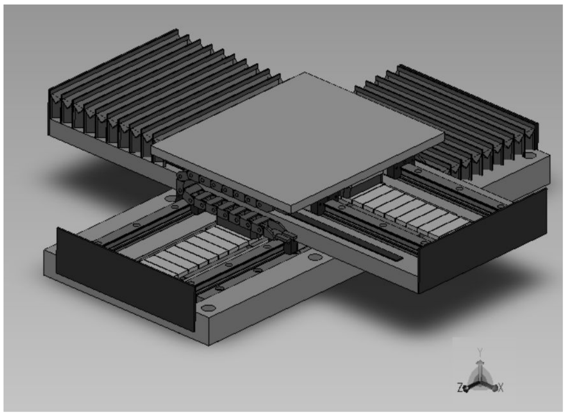

Fig.8. 3D CAD model of the table

Its technical parameters are arranged in Table III.

IV. CONCLUSION

The object of the present work is the design of a PMLSM for linear servo-motor driven XY table for laser marking. The design uses a complex approach in which the theory of electric and magnetic circuits is combined with FEM. This design is computer aided and therein a special script program was used. With the theory of electric and magnetic circuits the initial sizes of the magnetic system are calculated, then, based on iteration procedure using FEM simulations, the linear motor thrust is calculated. Its desired value serves as a criterion for the end of the iteration procedure. FEM was used also in the motor optimal design via the random optimization method.

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