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On one Mathematical Model for Dynamics of Propagation and Retention of Heat over New Fibre Insulation Coating

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Abstract. **In circumstances, when it is important to replace insulation materials with high content of emissions during production it is necessary to create new heat and sound insulation material, which eliminates CO_2 emissions, develop its production techniques and technological machinery – raw material chopper, pulp mixer, termopress, dryer chamber, formatting knifes, determine technical control parameters and control equipment, develop mathematical model of the material and calculation methods for design works. It is necessary to design, manufacture and experimentally test the respective technological equipment for insulation production pilot plant. To get exact physical parameters it is necessary design, manufacture and test unique laboratory equipment for determining the properties of insulation material. The mathematical model describing the dynamics of propagation and retention of heat over fibre insulation coating by taking "inner" specificities (graininess and porosity of layered structure of the considered fibre insulation) of heat insulator into account is proposed in the present paper.**

Keywords: **Insulation material, mathematical model, thermoelastic deformation and thermal movement, temperature distribution.**

I INTRODUCTION

As it is known (see, for instance, [1], [2] and respective references given in these), in the capacity of fibre material could be used wide range of materials, both organic and inorganic, for instance, cellulose, fabrics, wool, cotton, glass, rockwool, basalt fibre, etc. As the insulator could be used chopped polystyrene, polyurethane, cork, peat, bark, etc.

Fig. 1. Foamed polystyrene particles are bonded with cellulose fibers.

Binding together of the insulation particles, forming self-supporting layer of insulation material, useful both for thermal and for acoustic insulation (see, for instance, [3]). Remarkable positive property among others is ability of the material to accept and release water vapour – "breathe" like most of the natural materials. Other – it is stable against setting – opposite to pure cellulose wool insulation.

New insulation has been developed by the Liepaja University scientists. It is based on mix of insulation material particles enclosed in fibrous mass, having insulation properties (as it contains trapped air micro pockets) in the same time it.

The brief discussion on heat retention and energy conservation of the fibre insulation coating: fibre insulation coating can provide energy savings of 20-40% depending on ambient temperature, contents, weather conditions and application thickness. According to test data and results from applications (see, for instance, [4]-[8] and respective references given in these) efficiency is higher in conditions with exposure to convection-based cooling compared to uninsulated surfaces. Efficiency in convection-based cooling conditions is roughly comparable to conventional natural materials, but the difference is

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that coating-based insulated equipment will experience a slow heat drop and faster recovery compared to a slow heat drop and slow recovery in a conventionally insulated structure.

Fibre insulation coating relies primarily on radiant heat blocking while conventional material slows down heat transfer with dense mass. The coating's high saturation of insulation particles provides very low emissivity and transmittance characteristics that block radiative heat transfer. While there is some conductive heat transfer blocking, that's a smaller contribution compared to radiative blocking. A good analogy for low emissivity is "Low E" window coatings that also work by blocking solar radiation wavelengths' heat transfer. These two heat transfer characteristics are very important in understanding how the thin coating blocks transfer of radiant heat energy.

Conventional insulation material's heat transfer rating test is conducted at 0% humidity and 20°C. However, in the real world material becomes saturated with moisture at least up to the volume of relative humidity of surrounding air. Not a problem in very dry conditions, however, in most of the world that translates to 40-60% relative humidity and simply a matter of time before almost all mass-based insulations are saturated to the local relative humidity level. At only 30% moisture content, chopped polystyrene's heat conductivity is reduced almost to that of water at 60°C and very close to window glass at 80°C (see, for instance, [6], [9]). The advantage of coating is that it will not accept moisture and retains low heat transfer characteristics indefinitely.

And just as conventional insulation has a point of diminishing return when it comes to thickness; fibre insulation coating's effectiveness also becomes marginal beyond certain thicknesses at varying temperatures. Because radiant barriers react with thermal energy, they're useful when you have a heat source and do not store the heat energy to maintain a thermal battery as do mass-based conductive materials. This means that fibre insulation coating is very effective on heated equipment and much less so on unheated equipment applications such as preventing cold water lines from freezing. Insulation coating is effective in cold spaces with condensing surfaces based on it's low transmittance characteristics that essentially "take the chill off" and raise them above the dew point ([6], [7], [9]). Overall, insulation coating has many proven application in the field and can also be combined with conventional materials for solutions that leverage the best qualities of both.

II MATHEMATICAL MODEL FOR DYNAMICS OF PROPAGATION AND RETENTION OF HEAT OVER FIBRE INSULATION COATING

In this section we propose the mathematical model describing the dynamics of propagation and retention of heat over fibre insulation coating by taking "inner" specificities (graininess and porosity of layered structure of the considered fibre insulation; see, for instance, [5], [8], [10] as well as [11]) of heat insulator into account. It should be noted that the proposed model has its limitations: it describes only "internal" physical processes includes:

- heat propagation in the insulation material;
- mechanical process, related to tensions in material structure and differences in elasticity of said material under the influence of uneven heat spreading in the insulation material, which has been regarded as non-homogeneous layered structure.

Thus, the proposed mathematical model has the following statement:

1. Four equations concerning sought-for functions $T = T(x, y, z; t)$ and $u(x, y)$

$$\rho \cdot c \cdot \frac{\partial T}{\partial t} + \nabla_{x,y,z} \left\{ k \cdot \nabla_{x,y,z} T \right\} = f, \quad (x, y, z; t) \in \operatorname{int} D \times (0, t_{end}]; \tag{1}$$

$$2 \cdot \frac{\partial^2 u_{xx}}{\partial x^2} + \frac{\partial^2 u_{xx}}{\partial y^2} + \frac{\partial^2 u_{yy}}{\partial x^2} + \frac{\partial^2 u_{yy}}{\partial x^2} + E_l \cdot \alpha_l \cdot \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{\partial^2 T_0}{\partial x^2} - \frac{\partial^2 T_0}{\partial y^2} \right\} = 0, \tag{2}$$

$$(x, y, z; t) \in \operatorname{int} D \times (0, t_{end}];$$

$$\frac{\partial u_{xx}}{\partial x} + \frac{\partial u_{xy}}{\partial y} = 0, \ (x, y, z) \in \operatorname{int} D; \tag{3}$$

$$\frac{\partial u_{xy}}{\partial x} + \frac{\partial u_{yy}}{\partial y} = 0, \ (x, y, z) \in \operatorname{int} D; \tag{4}$$

2. Initial condition

$$T\big|_{t=0+0} = T_0(x, y, z), \ (x, y, z) \in D; \tag{5}$$

3. Eighteen boundary conditions concerning both the thermal field $T = T(x, y, z; t)$ (six boundary conditions) and the thermoelasticity $u(x, y)$ (twelve boundary conditions)

$$T\big|_{x=0+0} = T_{0x}(y, z; t), \quad (y, z; t) \in \left(D / \left[0, L_x\right]\right) \times \left[0, t_{end}\right]; \tag{6}$$

$$T\big|_{x=L_x-0} = T_{Lx}(y, z; t), \quad (y, z; t) \in \left(D / \left[0, L_x\right]\right) \times \left[0, t_{end}\right]; \tag{7}$$

$$T\big|_{y=0+0} = T_{0y}(x, z; t), \quad (x, z; t) \in \left(D / \left[0, L_y\right]\right) \times \left[0, t_{end}\right]; \tag{8}$$

$$T\big|_{y=L_y-0} = T_{Ly}(x, z; t), \quad (x, z; t) \in \left(D / \left[0, L_y\right]\right) \times \left[0, t_{end}\right]; \tag{9}$$

$$T\big|_{z=0+0} = T_{0z}(x,y;t),\quad (x,y;t) \in (D/[0,L_z]) \times [0,t_{end}]; \tag{10}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=L_z-0} = T_{Lz}(x,y;t),\quad (x,y;t) \in (D/[0,L_z]) \times [0,t_{end}]; \tag{11}$$

$$u_{xx}\big|_{x=0+0} = u_{xx}^{0x}(y),\ y \in [0,L_y]; \tag{12}$$

$$u_{xx}\big|_{x=L_x-0} = u_{xx}^{Lx}(y),\ y \in [0,L_y];$$

$$u_{xx}\big|_{y=0+0} = u_{xx}^{0y}(x),\ x \in [0,L_x]; \tag{13}$$

$$u_{xx}\big|_{y=L_y-0} = u_{xx}^{Ly}(x),\ x \in [0,L_x]; \tag{14}$$

$$u_{xy}\big|_{x=0+0} = u_{xy}^{0x}(y),\ y \in [0,L_y]; \tag{15}$$

$$u_{xy}\big|_{x=L_x-0} = u_{xy}^{Lx}(y),\ y \in [0,L_y]; \tag{16}$$

$$u_{xy}\big|_{y=0+0} = u_{xy}^{0y}(x),\ x \in [0,L_x]; \tag{17}$$

$$u_{xy}\big|_{y=L_y-0} = u_{xy}^{Ly}(x),\ x \in [0,L_x]; \tag{18}$$

$$u_{yy}\big|_{x=0+0} = u_{yy}^{0x}(y),\ y \in [0,L_y]; \tag{19}$$

$$u_{yy}\big|_{x=L_x-0} = u_{yy}^{Lx}(y),\ y \in [0,L_y]; \tag{20}$$

$$u_{yy}\big|_{y=0+0} = u_{yy}^{0y}(x),\ x \in [0,L_x]; \tag{21}$$

$$u_{yy}\big|_{y=L_y-0} = u_{yy}^{Ly}(x),\ x \in [0,L_x]. \tag{22}$$

In the proposed model (1)-(22) there are the following notations and assumptions:

- $D \overset{def}{\equiv} \{(x,y,z) : x \in [0,L_x], y \in [0,L_y], z \in [0,L_z]\}$ is geometric configuration (the closed 3D domain) of the insulation material that has a rectangular shape with the thickness / depth L_z;
- L_x and L_y are the length and the width of the rectangular insulation material, respectively;
- $\operatorname{int} D$ is an open domain that signifies interior of the domain D: $\operatorname{int} D \overset{def}{\equiv} D/\partial D$, ∂D contains the frontier points of the D;
- t_{end} is the time within a period of which we investigate the thermal processes occurring interior of the insulation material;
- the sought-for function $T = T(x,y,z;t)$ is the temperature (or rather the thermal field) in the considering point (x,y,z) of the insulation material at the time moment t;
- the prescribed function $k = k(x,y,z) > 0$ is the heat conductivity coefficient in the considering point (x,y,z) of the insulation material;
- the prescribed function $\rho = \rho(x,y,z) > 0$ is the density of the insulation material in the considering point (x,y,z) of the insulation material;
- the prescribed function $c = c(x,y,z) > 0$ is the specific thermal capacity of the insulation material in the considering point (x,y,z) of the insulation material;
- the prescribed function $f = f(x,y,z;t)$ is the power density of external heat sources applied to the considering point (x,y,z) of the heat insulator at the time moment t;
- the sought-for functions $u_{xx} = u_{xx}(x,y)$, $u_{xy} = u_{xy}(x,y)$ and $u_{yy} = u_{yy}(x,y)$ are the components of mechanical stress and thermoelasticity $u(x,y)$ under assumption of ignoring any changes / influences in the direction of the axis OZ;
- $\nabla_{x,y,z} T \overset{def}{\equiv} \frac{\partial T}{\partial x}\cdot\vec{e}_1 + \frac{\partial T}{\partial y}\cdot\vec{e}_2 + \frac{\partial T}{\partial z}\cdot\vec{e}_3$ is the gradient of thermal-vector field, where $\vec{e}_i$ $(i = \overline{1,3})$ are unit vectors located on the coordinate axes (OX, OY, OZ), respectively;
- the prescribed function $\alpha_l = \alpha_l(x,y,z) \overset{def}{\equiv} \frac{\Delta l}{l \cdot \Delta T}$ is the linear thermal expansion coefficient (see, for instance, [12]);
- the prescribed function $E_l = E_l(x,y,z) \overset{def}{\equiv} \frac{F \cdot l}{S \cdot \Delta l}$ is the modulus of elongation (so-called "Young modulus"; see, for instance, [13], [14]), which characterizes the deformation taking place in the considering point (x,y,z) of the heat insulator surface S under the impact of temperature voltage both sides relative to the heat insulator (i.e. from the outside and on the inside of premises): this Young modulus characterizes also the properties of the heat insulator to make resistance to tension at the elastic deformation under the impact of the temperature voltage;
- the prescribed function $T_0 = T(x,y,z;t)\big|_{t=0+0}$ is the initial temperature in the considering point (x,y,z) of the insulation material;

- the boundary functions $T_{0x}(y,z;t)$, $T_{Lx}(y,z;t)$, $T_{0y}(x,z;t)$, $T_{Ly}(x,z;t)$, $T_{0z}(x,y;t)$, $T_{Lz}(x,y;t)$, $u_{xx}^{0x}(y)$, $u_{xx}^{Lx}(y)$, $u_{xy}^{0y}(x)$, $u_{xy}^{Ly}(x)$, $u_{yy}^{0y}(x)$, $u_{yy}^{Ly}(x)$, $u_{xy}^{0x}(y)$, $u_{xy}^{Lx}(y)$, $u_{yy}^{0x}(y)$ and $u_{yy}^{Lx}(y)$ are assumed as known functions in their applicable / definitional domains.

In order to make sure that the system of four equations (1)-(4), which describes the interrelated processes generating temperature field $T(x,y,z;t)$ and thermoelasticity $u(x,y)$, had a physical determinacy (i.e. physical meaning), it is necessary to have some initial information of quantitative and qualitative patterns (see, for instance, [4], [5], [15]-[17] and respective references given in these). The initial condition (5) and the boundary conditions (6)-(22) form the required quantitative information. As regards the required information of qualitative pattern, its forming mostly depends on the chosen methods of analysis and solving the constructed model. Obviously, the less constrains are imposed on the model, the wider the range of application of this model becomes. Therefore, it makes sense to pose a question on finding the optimal set of constrains of qualitative pattern. However, in view of the fact that we cannot solve the proposed model (1)-(22) in present paper, the formulated below constrains of qualitative pattern are conditioned only by mathematical correctness of the equations (1)-(4):

- $T(x,y,z;t) \in C^{1,2}\left\{\operatorname{int} D \cup \left[0,t_{end}\right]\right\}$;
- $u(x,y) \in C^2\left\{\operatorname{int} D / \left(0,L_z\right)\right\}$;
- $T_0(x,y,z) \in C^2\{D\}$.

Thus, the proposed model (1)-(22) is the complete statement of the initial-boundary-value problem for investigation of the dynamics of propagation and retention of heat over fibre insulation coating by taking "inner" specificities of heat insulator into account. The analytical and / or numerical solution of the proposed model (1)-(22) will allow finding the sought-for functions $T(x,y,z;t)$ in the domain D and $u(x,y) \overset{def}{\equiv} \left(u_{xx}(x,y), u_{xy}(x,y), u_{yy}(x,y)\right)$ in the domain $D/\left[0,L_z\right]$, and consequently, in the time-interval $\left[0,t_{end}\right]$, during which the thermal processes occurring interior of the insulation material are investigated, we can completely determine the thermal field and the thermoelasticity of the considered insulation material having a "parallelepiped" shape with spatial measurements $L_x \times L_y \times L_z$.

III CONCLUSIONS

In this paper we formulated boundary conditions, among which only one is the second kind boundary condition (so-called the Neumann condition), while others are the Dirichlet boundary conditions. This is due to the practical point of view: as a rule an experimental method, which allows to realize boundary functions of the first kind, is an easier technique (however, such approach is not always expedient!). In addition, in present paper we formulated three conditions / constraints of qualitative pattern (one of many possible variants for constructing the qualitative constraints), the implementation of which together with the formulated boundary conditions and the initial condition unambiguously ensures mathematical correctness of the proposed model.

Qualitative analysis of the proposed model and / or its solving can favour for profound investigation and understanding of the thermal and thermomechanical processes occurring in the insulation materials, and thereby can improve the functionality and reliability of heat insulators.

To conclude with, let us note that authors of this paper are intended to continue the further investigation taking the benefit of both qualitative and quantitative studies for the proposed model (1)-(22) as well as to develop the stable analytical and numerical methods for their solution ensuring the corresponding computer-based implementation.

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