

On one Scientifically Based Sowing Management for Getting Pareto-optimal Crops Harvest

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Abstract - In the present work, we construct and study a mathematical model for one important agrarian problem of grain production, in which it is necessary to obtain such a guaranteed harvest of crops, the yield of which depends on soil-climatic conditions, so that the gross income from the sale of the grown crop is maximum. The constructed mathematical model is a multi-criteria optimization problem (with five criteria), and it can be attributed to optimal control, in which the controlled parameters are the kind and proportion of crops to be sown. Based on the results obtained, a concrete example is implemented using the application package Mathcad, version 14.0.0.163.

Keywords - Optimization problem, Pareto-optimal decision-making, guaranteed harvest, cereal crops

I. INTRODUCTION

For almost all countries in the world, agriculture is one of the most important branches of material production – growing cultivated plants and raising animals to provide population with food, and obtaining raw materials for needs of many industries. There are about 50 varieties of agriculture that can be divided into two groups: high-commodity agriculture and low-commodity agriculture. High-commodity agriculture includes: intensive farming and animal husbandry, gardening and horticulture, extensive steam and fallow farming, livestock farming;

low-commodity agriculture includes more backward plough and hoe-mattock farming, pastoralism, nomadic cattle breeding, as well as gathering, hunting and fishing. The countries of the European Union are characterized by high-commodity agriculture, which is achieved by a high level of mechanization and chemicalization, as well as by direct and indirect application of the combined achievements of a number of scientific fields, such as mathematics, physics, chemistry, biology, geology, botany, and economics. Currently, the agro-industrial complex in the highly developed countries of the European Union (Austria, Belgium, France, Germany, Italy, the Netherlands, Sweden) has acquired the form of agribusiness using agricultural SMART-machines, nanotechnologies and nano-materials, genetic engineering and biotechnology, robotics and electronics etc., which gives the agriculture of these countries a post-industrial character, whose unchanging sign is an extremely high level of intensification. In all other countries of the European Union, the agro-industrial complex has an industrial character with varying degrees of intensification. Developed agriculture is one of security factors of the European Union's countries – due to it food dependence is decreasing. For this reason, agriculture in the European Union is supported and subsidized in concordance with the Common Agricultural Policy. One of the main branches of

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agriculture is farming – the use of land for the purpose of growing crops, in particular, crops which will be discussed in this paper. Depending on soil and climatic conditions, farming is divided into the following categories: land reclamation (melioration farming); irrigated cropping; dry farming. Most countries of the European Union, including Latvia, have irrigation cropping/farming [1]. In turn, the section of agriculture devoted to the cultivation of cereal crops is called grain farming. Cereals – the most important group of crops – are the main human product, nutritious feed for farm animals, and raw materials for many industries [2]-[5].

The main indicators of soil fertility necessary for the formation of high yields of cereal crops are agrophysical indicators (basic: density, porosity, fine-grained structure, water-strength structure), biological indicators (basic: the presence of organic matter, including humus, phytosanitary state, biological activity, enzymatic activity) and agrochemical indicators (absorption capacity, soil reaction (pH), the presence of nutrients). Crop yields are very sensitive not only to soil fertility indicators, but also to climatic indicators, the main of which are temperature-humidity and temperature-wind (taking into account radiation) indicators. Together these indicators are called soil-climatic indicators (conditions), and it is these conditions that determine the success or failure of all stages of the process of growing cereal crops – the stage of selecting a variety of grain culture, its vernalization, the stages of processing and preparing the soil, the stages of determining the optimal sowing time and optimal seeding rates, stage of seed treatment, stages of seasonal treatment of crops, etc. [2], [3], [6], [7]. The yield of cereal crops by country, depending on soil and climatic conditions, crop farming, as well as macroeconomic conditions, is quite different. Even in the countries of the European Union, there are noticeable differences both in the yield of cereal crops and in the costs of growing them, and, consequently, in their costs to the final consumer [2], [8]-[11]. In addition, different crops respond differently to the same soil and climatic conditions that occur during a particular growing season. For example, wheat, rye, barley, and oats are grains of a temperate climate, however, with the same indicators of soil fertility, they differ greatly in terms of required climatic conditions; there are differences even between their winter and spring forms; these differences are significant also between kinds of crops when they adapt to certain soil and climatic conditions; winter rye tolerates the lowest temperatures, winter barley is the most sensitive cereal crop; winter wheat on this basis occupies a middle position. Winter crops tolerate lower temperatures when they gradually harden. Changing temperatures close to the freezing point cause enzymatic activity inside the cells, which reduces their cold resistance. Highly developed or early-growing crops are especially sensitive to this. The danger of death is high in crops that are affected by diseases, pests, birds or sudden onset of cold during intense metabolic processes. This is usually observed at the beginning of winter or during spring frosts. In addition, the death of winter crops from freezing is caused not only by mechanical destruction of the cells by ice, but also by bulging/drying of the sprouts of crops. It is caused by a

change of negative night temperatures and positive daytime temperatures, and as a result of soil movement, root hairs or even skeletal roots break off, and the sprouts themselves appear to be squeezed out of the soil.

In this work, we consider the problem of obtaining guaranteed harvest of cereal crops that does not depend on soil and climatic conditions. The importance of studying such a problem, among other things, is due to global climate changes that have been observed in recent years. The verbal statement of the considered problem is given in the next section. The authors of this work, having analysed more than 200 publications on crops growing published in international journals of various levels over the past 25 years, did not find a similar statement of the problem: there are many publications on obtaining guaranteed harvest (chiefly, single-criterion optimality), and in the majority of these publications, statistical approaches are used, which causes results to have probabilistic nature.

II. VERBAL FORMULATION OF THE CONSIDERING AGRARIAN PROBLEM

Let us suppose that some agricultural enterprise is going to sow N sown fields with M kinds of crops, yield of which depends on K types of soil-climatic conditions [1]-[7] (we will call these conditions climatic scenarios). According to the procurement contract (see Remark 1) between an agricultural enterprise and a procurer (for example, a state) that purchases produced crops for further processing and/or sale, the agricultural company is obliged to sell to the procurer cereals of m -th ($m = \overline{1, M}$) kind in amount of not less than $\underline{Q}_m^{p.c.} \geq 0$ quintals for the purchase price $p_m^{p.p.}$ (see Remark 3 as well as [8]-[10]). Provided that the demand for each of the produced crops is unlimited, it is necessary to determine what kinds of crops and in what proportions should be sown in order, firstly, to obtain a guaranteed crop (the maximum of the minimum possible) that does not depend on climatic scenarios, and secondly, the gross income (see Remark 2) from the sale of the crop would be the largest?

Remark 1. A procurement contract is a type of contract of sale, and is an agreement governing relations associated with the procurement from agricultural organizations and peasant farms of agricultural products grown or produced by them. In accordance with the contract agreement, the agricultural producer agrees to transfer the agricultural products grown or produced by him to the buyer-procurer (for example, the state), who purchases such products for further processing and/or sale. In the considered problem absence of a procurement contract between an agricultural enterprise and a procurer regarding any crop of the kind m ($m = \overline{1, M}$) means that one needs to put $\underline{Q}_m^{p.c.} = 0$; if such an agreement does not take place at all, then, obviously, $\underline{Q}_m^{p.c.} = 0$ for $\forall m = \overline{1, M}$. End of Remark (EOR)

Remark 2. Gross income is the income that the company receives from its core business, as well as from interest, dividends or royalties that other companies pay

them. The sum of gross income, as a rule, refers to all gross income or gross income for a certain period of time in a certain currency. EOR

Remark 3. The purchase price is the type of wholesale price used in the procurement of agricultural products by procurers (for example, the state) in the domestic market. Purchase prices are differentiated depending on the quality of products, taking into account the geographical segmentation of the market, and are defined as the price of agricultural products purchased by procurers from producers under contract agreements. EOR

Let us introduce the following notation for the original data of the above formulated verbal problem: S_n is area (ha) of n -th ($n = \overline{1, N}$) sown field; $q_{n,m,k}$ is yield (quintal/ha) of cereal crop of m -th ($m = \overline{1, M}$) kind, grown under k -th ($k = \overline{1, K}$) climatic scenario in n -th ($n = \overline{1, N}$) sown field; $p_{m,k}^{p.p.}$ is purchase price (euro) of 1 quintal of cereal crop of m -th ($m = \overline{1, M}$) kind, grown under k -th ($k = \overline{1, K}$) climatic scenario; $p_{m,k}^{m.p.}$ is purchase price (euro) of 1 quintal of cereal crop of m -th ($m = \overline{1, M}$) kind, grown under k -th ($k = \overline{1, K}$) climatic scenario. In this work we will assume that the purchase price $P^{\text{purchasing price}} = \{p_{m,k}^{p.p.}\}_{m=1, \overline{M}}^{k=1, \overline{K}}$ of the grown cereal crops, as opposed to their market price $P^{\text{market price}} = \{p_{m,k}^{m.p.}\}_{m=1, \overline{M}}^{k=1, \overline{K}}$, does not depend on the climatic scenarios, i.e. we will assume that $p_{m,k}^{p.p.} \equiv p_m^{p.p.}$ for $\forall k = \overline{1, K}$.

Remark 4. The assumption about dependence of the market price of grown cereal crops on climatic scenarios may seem absurd at first glance: after all, quality and price of the products are important to customers, not soil and climatic conditions under which crops were grown, or the difficulties that the manufacturer had to overcome when growing crops. Of course, if we consider a small country (for example, Latvia) with almost the same climatic conditions and relatively uniform soil characteristics, then the assumption that the market price of cereal crops is independent of soil and climatic conditions under which they were grown would be reasonable. However, in our opinion, for some countries with quite sharp climatic and soil differences, the assumption made has the right to exist: in this work, when constructing a mathematical model and its subsequent research, we will proceed from this assumption, however, the results obtained can easily be adapted to the case when the market price of some or all of the grown crops does not depend on climatic scenarios – for this it is necessary (and sufficient) to put in the mathematical model that $p_{m,k}^{m.p.} = p_m^{m.p.}$ for $\forall k = \overline{1, K}$, as it is done in the example considered in this paper. EOR

Remark 5. It is obvious that instead of N sown fields one could consider only one field. Then, nothing fundamental would have changed: only instead of N areas $\{S_n\}$ there would be one common area S of the sown field and instead of three-index yields $\{q_{n,m,k}\}$ there would be two-index $\{q_{m,k}\}$. However, it seems to us that considering N areas is rational in the sense that an agricultural enterprise may have sown fields that are geometrically quite distant from each other (they may be located in different regions of the country, or even in different countries) and, therefore, it will be more convenient for interested people of the corresponding profile to use the results of this work that can be easily programmed on computers. EOR

III. MATHEMATICAL MODEL OF THE VERBALLY FORMULATED AGRARIAN PROBLEM, AND ITS INVESTIGATION

In order to construct a mathematical model of the problem formulated in the previous section, we denote by $x_{n,m}$ the required area of n -th ($n = \overline{1, N}$) field, planted with crops of m -th ($m = \overline{1, M}$) kind, and denote by V the required guaranteed total volume of crops grown (which must be maximized) for any of K climatic scenarios, i.e. $V \rightarrow \max$. Then using the introduced designations of the original data, we can write that $\sum_{m=1}^M q_{n,m,k} x_{n,m}$ is the volume of all crops grown at k -th ($k = \overline{1, K}$) climatic scenario on n -th ($n = \overline{1, N}$) field. Therefore, the following NK inequalities and N equalities must hold:

$$\sum_{m=1}^M q_{n,m,k} \cdot x_{n,m} \geq V, \quad n = \overline{1, N}, \quad k = \overline{1, K};$$

$$\sum_{m=1}^M x_{n,m} = S_n, \quad n = \overline{1, N}.$$

Further, under the assumption that during the period of sowing and growing crops there was k -th ($k = \overline{1, K}$) climatic scenario, yield of crops of m -th ($m = \overline{1, M}$) kind, grown in all N fields, is equal to $\sum_{n=1}^N q_{n,m,k} x_{n,m}$. Then, it is obvious that the implementation of the procurement contract between the agricultural enterprise and the procurer requires that the following $N \cdot K$ inequalities hold:

$$\sum_{n=1}^N q_{n,m,k} x_{n,m} \geq Q_m^{p.c.}, \quad m = \overline{1, M}, \quad k = \overline{1, K}.$$

It is also obvious that the company's income from the sale of crops of m -th ($m = \overline{1, M}$) kind, grown in all N fields under k -th ($k = \overline{1, K}$) climatic scenario, is equal to

$$p_{m,k}^{m.p.} \cdot \left(\sum_{n=1}^N q_{n,m,k} x_{n,m} - Q_m^{p.c.} \right) + p_m^{p.p.} \cdot Q_m^{p.c.}.$$

Therefore

$$\sum_{m=1}^M \left(p_{m,k}^{m.p.} \cdot \left(\sum_{n=1}^N q_{n,m,k} x_{n,m} - Q_m^{p.c.} \right) + p_m^{p.p.} \cdot Q_m^{p.c.} \right)$$

is the gross income of the enterprise from the sale of all M cereal crops, grown in all N fields under the conditions of k -th ($k = \overline{1, K}$) climatic scenario. The requirement of maximality of gross income:

$$\sum_{n=1}^N \sum_{m=1}^M p_{m,k}^{m.p.} q_{n,m,k} x_{n,m} \rightarrow \max, k \in \overline{1, K}.$$

Here we omitted the constant $\sum_{m=1}^M Q_m^{p.c.} (p_m^{p.p.} - p_{m,k}^{m.p.})$ due to the fact that it does not play any role in maximizing the gross income function.

Let's introduce new variables $y_j = x_{n,m}$, where:

- $y_{NM+1} = V$;
 - for each fixed ordered couple (n, m) , in which $n \in \{1, \dots, N\}$ and $m \in \{1, \dots, M\}$, index j is calculated by the formula
- $$j = M(n-1) + m, j \in \{1, \dots, NM\};$$
- for each fixed index $j \in \{1, \dots, NM\}$ indices $n \in \{1, \dots, N\}$ and $m \in \{1, \dots, M\}$ are determined uniquely by the formulas [12]

$$m \equiv j \pmod{M},$$

$$n = 1 + \frac{j-m}{M}.$$

Then, by taking the new variables $y = \{y_j\}_{j=1, (NM+1)}$ into account, combining the results obtained above, we can formulate the following multicriteria optimization problem, which is a mathematical model of the considered agrarian problem

$$\left\{ \begin{array}{l} \text{maximize } \left\{ w_1(y) \equiv y_{NM+1} \right\}, \\ \text{maximize } \left\{ w_{k+1}(y) \equiv \sum_{j=1}^{NM} \xi_{j,k} y_j \right\}, k \in \overline{1, K}, \\ \sum_{m=1}^M q_{n,m,k} y_{M(n-1)+m} \geq y_{NM+1}, n = \overline{1, N}, k = \overline{1, K}, \\ \sum_{n=1}^N q_{n,m,k} y_{M(n-1)+m} \geq Q_m^{p.c.}, m = \overline{1, M}, k = \overline{1, K}, \\ \sum_{m=1}^M y_{M(n-1)+m} = S_n, n = \overline{1, N}, \\ y_j \geq 0, j = \overline{1, NM}, y_{NM+1} > 0. \end{array} \right. \quad (1)$$

where $\xi_{j,k} = \xi_{M(n-1)+m,k} = p_{m,k}^{m.p.} q_{n,m,k}$.

Remark 6. In the mathematical model (1), the number of unknown variables is $NM + 1$, and all these variables are involved in all K different climatic scenarios, i.e. we have not added an index k ($k = \overline{1, K}$) to these variables. It is not a mistake: this is due to the fact that different soil and climatic conditions (climatic scenarios) cannot simultaneously take place for the same sown field. Let us show this using the following simple example [13], for which the mathematical model (1) is greatly simplified and turns into a one-criterion linear programming problem. Let $N = 1$ (one sown field with area S), $M = 4$ (four kinds of cereal crops), $K = 2$ (two climatic scenarios); yields of cereal crops (quintal/ha): $q_{1,1,1} = 25$; $q_{1,2,1} = 20$, $q_{1,3,1} = 30$, $q_{1,4,1} = 15$, $q_{1,1,2} = 15$, $q_{1,2,2} = 20$, $q_{1,3,2} = 10$, $q_{1,4,2} = 40$. It is required to determine what kinds of crops and in what proportion should the sown field be sown in order to obtain the maximum guaranteed yield, regardless of which of the two climatic scenarios will take place. As mentioned above, for this simple problem, the mathematical model (1) is simplified to a one-criterion optimization model and has the following form:

$$\text{maximize } \left[w(x) \equiv y_5 \right]$$

subject to

$$\left\{ \begin{array}{l} 25y_1 + 20y_2 + 30y_3 + 15y_4 - y_5 \geq 0, \\ 15y_1 + 20y_2 + 10y_3 + 45y_4 - y_5 \geq 0, \\ y_1 + y_2 + y_3 + y_4 = 1, \end{array} \right.$$

where $y_j \geq 0$ is the area of the field sown with crop of j -th ($j = \overline{1, 4}$) kind. Solving this problem with Danzig's simplex algorithm, we find: $y_1 = 0$, $y_2 = 0$, $y_3 = \frac{5}{9}S$, $y_4 = \frac{4}{9}S$, $y_5 = \frac{70}{3}S$. In other words, we obtain that the maximum guaranteed yield is equal to $\frac{70}{3}S$, which is achieved only if the field is sown only with crops of the 3rd

and 4th kinds in proportions 5:4. The reader can independently verify that any other sowing plan will give a worse result. EOR

Model (1) is a linear programming $(K+1)$ - criteria problem, therefore, speaking of the solution of model (1), it should be understood as its Pareto-optimal solution, which has the property that any deviation from this solution gives rise to a situation where an improvement in the value of any of the criteria worsens the values of the remaining criteria [14]-[16]. In other words, the Pareto-optimal solution is a trade-off decision: each of the criteria strives to achieve its optimum (maximum or minimum) "while watching the reaction" of the corresponding optimums of all remaining criteria so that they do not deteriorate. Currently, to find the Pareto-optimal solution to the multicriteria optimization problem, there are many different approaches and methods that differ significantly in both idea/concept and implementation complexity [14]-[20]. In the works [12], [21] briefly, but exhaustively from the point of view of the application skill, three main methods for solving the multicriteria optimization problem are described – the weighted sum approach method, the epsilon-constraint method, and the goal attainment method of Gembicki. The paper [22] (see also relevant references given there) discusses in detail the main drawback of the weighted sum approach associated with the lack of scientifically based and objective selection of criteria importance coefficients (these coefficients are also called criteria weights), with which multicriteria optimization problem is reduced to a single-criterion optimization problem with the same limitations of the original multicriteria problem and with one convoluted criterion

$$w(y) = \sum_{k=1}^{K+1} \lambda_k w_k(y), \quad \text{where } \lambda_k > 0 \text{ is weighting}$$

coefficient of k -th $(k = \overline{1, K+1})$ criterion of the original multicriteria problem, and $\sum_{k=1}^{K+1} \lambda_k = 1$. For example: (a) if

we assume that for the decision maker in model (1) all the criteria are equally important, then instead of the multicriteria model (1) we get a single-criterion model with

$$\text{convoluted criterion } w(y) = y_{NM+1} + \sum_{k=1}^K \sum_{j=1}^{NM} \xi_{j,k} y_j, \text{ which}$$

we want to maximize (here we omitted the multiplicative constant $(K+1)^{-1}$ in the right-hand side of $w(y)$ since it does not play any role in maximizing the criterion) when all the same limitations of model (1) are fulfilled, i.e. all $(N+M)K+N$ essential constraints and all $NM+1$ sign constraints; (b) if we assume that for the decision maker in model (1), the importance of the criterion of maximality of guaranteed total volume of grown crops (in any climatic scenario) is p_{pct} % of the cumulative importance of all the criteria, and the importance of the gross income criterion of the enterprise from the sale of all grown crops in all fields, remains unchanged for any of the climatic scenarios that have occurred, then instead of the multicriteria model

(1) we get a single-criterion model with a convoluted

$$\text{criterion } w(y) = p_{pct} y_{NM+1} + \frac{100 - p_{pct}}{K} \sum_{k=1}^K \sum_{j=1}^{NM} \xi_{j,k} y_j,$$

which we want to maximize (here we omitted the multiplicative constant) 0.01 in the right-hand side of $w(y)$ since it does not play any role in maximizing the criterion) when all the same limitations of model (1) are fulfilled. In particular, if $p_{pct} = 50\%$, then

$$w(y) = y_{NM+1} + \frac{1}{K} \sum_{k=1}^K \sum_{j=1}^{NM} \xi_{j,k} y_j. \text{ Obviously, for different}$$

sets of weights $\{\lambda_k\}_{k=1, (K+1)}$ the corresponding single-criterion problems, to which the original single-criterion problem is reduced, may have different solutions and, moreover, some of them may not have solutions due to the unlimitedness of the minimized criterion from below (if we look for a minimum) or above (if look for a maximum). In the work [22], for finding for finding the weighting coefficients of criteria a new approach based on methods of the theory of inverse and ill-posed problems is proposed. However, for successful application of the proposed approach, special knowledge is needed – knowledge of the theory of inverse and ill-posed problems, which significantly limits the proposed approach.

The epsilon-constraint method, first proposed in [23], has its main insurmountable drawback associated with the lack of an established application procedure, namely, the researcher almost never knows exactly which of the criteria to translate into restrictions, and what values of epsilon to set on the right side of these restrictions [12]. These questions are subjectively decided by the researcher, depending on how much he understands the meaning of the task as a whole and the importance of each criterion in particular. Nevertheless, the epsilon-constraint method has gained some popularity due to the fact that it is very simple and straightforward, and it uses standard mathematical software for computer implementation.

The essence of the goal attainment method of Gembicki, first proposed in [17], is as follows: (a) all the criteria of the original multicriteria problem must be transformed so that their minimization or maximization is required (this is easy to do by multiplying the criteria by -1); (b) one should generate set of desired intentions $\{w_1^*; \dots; w_{K+1}^*\}$, which is related to criterion vector $(w_1(y), \dots, w_{K+1}(y))$ of the original multicriteria problem (1), for example, as a desired intention w_k^* one can take the optimal value of the corresponding single-criterion problem with the criterion $w_k(y)$ and with all the limitations of the original multicriteria problem (1); (c) a single-criterion problem should be solved in which it is required to find such a minimum value of a numerical parameter $R \in \mathbb{R}^1$, so that new constraints $w_k(y) - \lambda_k R \leq w_k^*, k = \overline{1, K+1}$ hold together (here

$\{\lambda_k \geq 0\}_{k=1, \overline{K+1}}$ are weighting coefficients that determine how close each criterion is to its target value) and all the limitations of the original multicriteria problem (1) are satisfied. In the new limitations of the goal attainment method of Gembicki, the value $\lambda_k R$ ($\sim \Delta w_k = w_k(y) - w_k^*$) can be interpreted as the degree of under-attainment/over-attainment of k -th ($k = \overline{1, K+1}$) desired intention w_k^* . In other words, $\lambda_k R, k = \overline{1, K+1}$ determine the rigidity of the desired intentions $w_k^*, k = \overline{1, K+1}$. For example [21], [24], [25], if the desired intention w_k^* is unattainable/overattainable, then a small value of the weighting coefficient λ_k will result in the degree of under-attainment/over-attainment Δw_k being small. Finally, let us note that in the case of under-attainment of the desired intentions, the smaller weighting coefficient is associated with the more important criteria, and in the case of over-attainment of the desired intentions, the smaller weighting coefficient is associated with the less important criteria).

In this paper, we will solve the mathematical model (1) by the goal attainment method of Gembicki, the brief essence of which has just been described above. So, the procedure for solving model (1) consists of the following steps.

Step 1. By any standard method (for example, Dantzig's simplex algorithm), we solve the following $K+1$ single-criterion linear programming problems:

$$\begin{aligned} & \text{minimize}_{y \in \Omega} (\tilde{w}_1(y) = -y_{NM+1}); \\ & \text{minimize}_{y \in \Omega} \left(\tilde{w}_{k+1}(y) = -\sum_{j=1}^{NM} \xi_{j,k} y_j \right), k = \overline{1, K}, \end{aligned}$$

where the set Ω , called a feasible region, is

$$\begin{aligned} \Omega \stackrel{\text{def}}{=} \{ & y \in \mathbb{R}_+^{NM}, y_{NM+1} > 0; \\ & \sum_{m=1}^M q_{n,m,k} y_{M(n-1)+m} \geq y_{MN+1}, n = \overline{1, N}, k = \overline{1, K}; \\ & \sum_{n=1}^N q_{n,m,k} y_{M(n-1)+m} \geq Q_m^{\text{p.c.}}, m = \overline{1, M}, k = \overline{1, K}; \\ & \sum_{m=1}^M y_{M(n-1)+m} = S_n, n = \overline{1, N} \}. \end{aligned}$$

As a result of this step, we obtain a set $\tilde{w}_k^{\min} = \min_{y \in \Omega} \tilde{w}_k(y), k = \overline{1, K+1}$, which we will use as the set of desired intentions $\{w_1^*; \dots; w_{K+1}^*\}$, i.e. $w_k^* = \tilde{w}_k^{\min}, k = \overline{1, K+1}$.

Step 2. Solve the following single-criterion problem:

$$\begin{cases} \text{minimize}(R), \\ R \in \mathbb{R}^1, y \in \Theta[R], \\ \Theta[R] = \{y \in \Omega: \tilde{w}_k(y) - \tilde{w}_k^{\min} R \leq \tilde{w}_k^{\min}\} \\ \forall k \in \overline{1, K+1}, \end{cases} \quad (2)$$

where as weighting coefficients $\{\lambda_k\}_{k=1, \overline{K+1}}$, which, as stated in the description of the goal attainment method of Gembicki, determine how close each criterion will be to its goal value, we took the desired intentions $\tilde{w}_k^{\min}, k = \overline{1, K+1}$. This means that we want to achieve the same measure of underattainability or overattainability of all $K+1$ criteria.

The results of solving problem (2) are the number $R^{\min} = \min_{R \in \mathbb{R}^1, y \in \Theta[R]} (R)$ and numerical vector

$$y^{\text{Pareto}} = (y_1^{\text{Pareto}}, \dots, y_{NM+1}^{\text{Pareto}})^T = \arg \min_{R \in \mathbb{R}^1, y \in \Theta[R]} (R),$$

the last coordinate of which, we recall, is the desired maximum guaranteed volume (the maximum of the minimum possible) of all M kinds of cereal crops grown in all N fields under any of K climatic scenarios, i.e. $y_{NM+1}^{\text{Pareto}} = V_{\max}^{\text{guaranteed}}$. The remaining coordinates of the

resulting numerical vector y^{Pareto} characterize the areas of fields sown with M kinds of cereal crops: $y_{M(n-1)+m}^{\text{Pareto}} = x_{n,m}^{\text{Pareto}}, n = \overline{1, N}, m = \overline{1, M}$.

So, doing the above two steps gives us a number $V_{\max}^{\text{guaranteed}}$, which is the desired guaranteed yield, and a

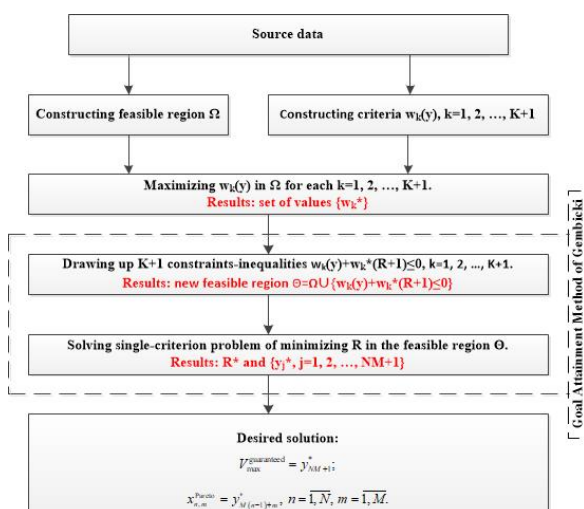
numerical matrix $x^{\text{Pareto}} = \{x_{n,m}^{\text{Pareto}}\}_{n=1, \overline{N}}^{m=1, \overline{M}}$, the elements of

which are the required areas for sowing, namely, the element $x_{n,m}^{\text{Pareto}}$ is the area of n -th field, which must be sown with cereals of m -th kind. It remains only to note that since the set $\{R^{\min}; y^{\text{Pareto}}\}$ found by the solution of

the single-criterion optimization problem (2) is Pareto-optimal of the multicriteria optimization problem (1) (this statement follows from the theoretical justification of the goal attainment method of Gembicki), then set

$\{V_{\max}^{\text{guaranteed}}; x^{\text{Pareto}}\}$ is Pareto-optimal solution of the agrarian problem that we are studying, i.e. a compromise solution in which a compromise occurs between the guaranteed volume of the crop (the maximum of the minimum possible) and the total gross income from the sale of the crop.

The approach described above can be conceptually represented in the form of the following diagram:



IV. NUMERICAL EXPERIMENT: IMPLEMENTATION OF MATHEMATICAL MODEL

In this section, using a specific example, we illustrate the application of the proposed mathematical model (1) and the chosen goal attainment method of Gembicki to find the Pareto-optimal solution of the considered agrarian problem.

Suppose that the agricultural enterprise "Latvijas labība" Ltd. is going to sow seven sown fields ($N = 7$) in the Jelgava region of Latvia with five kinds of cereals ($M = 5$) – bread wheat (#1), malting barley (#2), common buckwheat (#3), bread rye (#4) and milling oats (#5), the yield of which depends on four types of climatic scenarios ($K = 4$) – warm dry weather (#1), chilly dry weather (#2), warm rainy weather (#3), chilly rainy weather (#4), – which may occur during sowing and growing [1] of these cereal crops. It is assumed that in the corresponding time period in Latvia the demand for each of the above crops will be so high that it can be considered unlimited. According to the procurement contract between the agricultural enterprise "Latvijas labība" and the procurer "Zelta Dzirnavas" Ltd., which purchases the produced crops for further processing, the agricultural enterprise "Latvijas labība" is obliged to sell the bread wheat in amount of not less than 45 quintals, malting barley in amount of not less than 20 quintals, common buckwheat in amount of not less than 25 quintals, milling oats in amount of not less than 20 quintal. It is required to determine what kinds of crops listed above and in what proportions should be sown in order, firstly, to obtain a guaranteed crop (the maximum of the minimum possible) that does not depend on climatic scenarios, and, secondly, the gross income from the sale of the crop was the greatest? The necessary input data are given below (the indicated data are quite realistic data based on the corresponding official statistics on Latvia extracted from the sources [8]-[11] in the period of 2010-2018):

- area of sown fields (ha):

$$S = (20, 10, 15, 25, 40, 30, 60);$$

- purchasing and market prices of grown crops per a quintal (euro):

$$p^{p.p.} = (17.3, 13.2, 22.6, 10.2, 17.4),$$

$$p^{m.p.} = (18.6, 15.9, 28, 13.6, 21);$$

- bread wheat yield (quintal /ha):

$$q_{4 \times 7}^{(1)} = \begin{pmatrix} 30 & 25 & 35 & 31 & 29 & 31 & 27 \\ 25 & 20 & 30 & 26 & 24 & 26 & 22 \\ 45 & 40 & 50 & 46 & 44 & 46 & 39 \\ 35 & 30 & 40 & 36 & 34 & 36 & 31 \end{pmatrix};$$

- malting barley yield (quintal/ha):

$$q_{4 \times 7}^{(2)} = \begin{pmatrix} 20 & 22 & 21 & 22 & 20 & 22 & 19 \\ 30 & 35 & 35 & 34 & 32 & 34 & 29 \\ 25 & 21 & 23 & 24 & 22 & 24 & 21 \\ 25 & 19 & 19 & 22 & 20 & 22 & 19 \end{pmatrix};$$

- common buckwheat yield (quintal/ha):

$$q_{4 \times 7}^{(3)} = \begin{pmatrix} 8 & 7 & 7 & 8 & 7 & 8 & 7 \\ 10 & 11 & 10 & 11 & 9 & 11 & 11 \\ 9 & 9 & 8 & 9 & 10 & 10 & 8 \\ 9 & 9 & 9 & 10 & 8 & 10 & 9 \end{pmatrix};$$

- bread rye yield (quintal/ha):

$$q_{4 \times 7}^{(4)} = \begin{pmatrix} 20 & 25 & 22 & 23 & 22 & 23 & 20 \\ 40 & 42 & 42 & 42 & 40 & 42 & 36 \\ 35 & 22 & 28 & 29 & 28 & 29 & 25 \\ 30 & 38 & 30 & 33 & 32 & 34 & 29 \end{pmatrix};$$

- milling oats yield (quintal/ha):

$$q_{4 \times 7}^{(5)} = \begin{pmatrix} 10 & 9 & 9 & 10 & 8 & 10 & 9 \\ 15 & 13 & 14 & 15 & 13 & 15 & 13 \\ 20 & 25 & 20 & 22 & 21 & 23 & 19 \\ 25 & 30 & 27 & 28 & 27 & 28 & 24 \end{pmatrix}.$$

Now let us establish the following correspondences between indices $n = \overline{1, 7}$, $m = \overline{1, 5}$, $k = \overline{1, 4}$ of the designations in the model (1) and the names present in the considered illustrative case study (names of crops, names of climatic scenarios): index $n = \overline{1, 7}$ will correspond to the number of the sown fields; index $m = \overline{1, 5}$ will correspond to the number of considered cereal crops, at that $m = 1$ will correspond to bread wheat, $m = 2$ – to malting barley; $m = 3$ – to common buckwheat; $m = 4$ – to bread rye; $m = 5$ – to milling oats; index $k = \overline{1, 4}$ will correspond to climatic scenarios, at that $k = 1$ will

correspond to warm dry weather, $k = 2$ – to chilly dry weather, $k = 3$ – to warm rainy weather, $k = 4$ – to chilly rainy weather. Further, variable V stands for sought-for guaranteed crop that does not depend on climatic scenarios; variable $x_{n,1}$ stands for share of area of n -th field sown with bread wheat; variable $x_{n,2}$ stands for share of area of n -th field sown with malting barley; variable $x_{n,3}$ stands for share of area of n -th field sown with common buckwheat; variable $x_{n,4}$ stands for share of area of n -th field sown with bread rye; variable $x_{n,5}$ stands for share of area of n -th field sown with milling oats. Now the correspondence between the variables V , $\{x_{n,m}\}_{n=\overline{1,7}}^{m=\overline{1,5}}$ and the variables $\{y_j\}_{j=\overline{1,36}}$ of the model (1) is quite obvious: $V = y_{36}$, $x_{n,m} = y_{5(n-1)+m}$, $n = \overline{1,7}$, $m = \overline{1,5}$.

So, for the considered illustrative computational example, the mathematical model (1) takes the following form:

$$\text{maximize} \left\{ w_0(y) \stackrel{\text{def}}{=} y_{36} \right\}, \quad (3)$$

$$\text{maximize} \left\{ w_k(y) \stackrel{\text{def}}{=} \sum_{n=1}^7 \sum_{m=1}^5 p_m^{m.p.} \left(q_{4 \times 7}^{(m)} \right)_{k,n} y_{5(n-1)+m} - \sum_{m=1}^5 Q_m^{\text{p.c.}} \left(p_m^{\text{p.p.}} - p_m^{m.p.} \right) \right\}, k = \overline{1,4}, \quad (4)$$

subject to

$$\begin{cases} \sum_{m=1}^M \left(q_{4 \times 7}^{(m)} \right)_{k,n} y_{5(n-1)+m} \geq y_{36}, n = \overline{1,7}, k = \overline{1,4}, \\ \sum_{n=1}^7 \left(q_{4 \times 7}^{(m)} \right)_{k,n} y_{5(n-1)+m} \geq Q_m^{\text{p.c.}}, m = \overline{1,5}, k = \overline{1,4}, \\ \sum_{m=1}^5 y_{5(n-1)+m} = S_n, n = \overline{1,7}, \\ y_j \geq 0, j = \overline{1,35}, y_{36} > 0, \end{cases} \quad (5)$$

where $Q^{\text{p.c.}} = (45, 20, 25, 0, 20)$.

Now we apply the goal attainment method of Gembicki described in the previous section to the constructed five-criterion optimization problem, i.e. we carry out Steps 1 and 2.

Step 1. To specify a set of desired intentions we have to solve the following five single-criterion optimization problems: problem (3), (5); problem (4), (5) for $\forall k = \overline{1,4}$. Having solved these single-criterion linear programming problems by Danzig's simplex algorithm, we find the following set of desired intentions:

$$\begin{aligned} \{w_k^*\}_{k=\overline{0,4}} &= \left\{ \arg \max_{y: (6)} \{w_k(y)\} \right\}_{k=\overline{0,4}} \\ &= \{250; 108676.5; 107345.4; 161128.5; 126880.5\}. \end{aligned}$$

Step 2. We have to solve the following single-criterion linear programming problem:

$$\text{minimize}_{R \in \mathbb{R}^1, y \in \Theta[R]} [R], \quad (6)$$

subject to

$$\begin{cases} w_k(y) - w_k^* R \geq w_k^*, k = \overline{0,4}, \\ \text{Constraints (5)}, \\ R \in \mathbb{R}^1. \end{cases} \quad (7)$$

In the framework of this work, the problem (6), (7) was solved by us of the Tikhonov regularization method [26], implemented by the application package Mathcad, version 14.0.0.163. When applying this method, the optimal regularization parameter was found by two methods, both using the generalized residual principle [27] and using the method first proposed and substantiated in [28] (see also [22]): the results obtained coincide with an accuracy not exceeding 10^{-4} .

So, as a result of steps 1 and 2, we obtained the following results: $x_{1,1} = 20$, $x_{2,2} = 1.35$, $x_{2,4} = 8.65$, $x_{3,1} = 15$, $x_{4,1} = 25$, $x_{5,1} = 11.45$, $x_{4,4} = 28.55$, $x_{6,1} = 30$, $x_{7,1} \approx 53$, $x_{7,3} \approx 4$, $x_{7,5} \approx 3$. In other words, the first, third, fourth, and sixth sown fields should be sown only with bread wheat; the second sown field should be sown with malting barley and bread rye in a ratio of approximately 1:6; the fifth sown field should be sown with bread wheat and bread rye in a ratio of 2:5 approximately; finally, the seventh sown field should be sown with three cereal crops – bread wheat, common buckwheat and milling oats – in a ratio of approximately 99:5:7.

Thus, we can summarize that the agricultural enterprise "Latvijas labība" grows bread wheat on about 154.5 ha of its arable lands of 200 hectares, malting barley – on about 1.5 ha, common buckwheat – on about 4 ha, bread rye – on about 37 ha, and milling oats – on approximately 3 ha. As a result, the volume of the guaranteed maximum harvest of all 5 cereal crops, regardless of which of the four climatic conditions occurs, is approximately 7205 quintals (this is at least, and the maximum possible volume is approximately 94666 quintal), selling of which, taking into account the procurement contract, brings the enterprise approximately 128170 euros (accordingly, this value is the minimum guaranteed income and it can increase to 168441 euros).

In conclusion, we just add that the analysis of the results shows that, within the framework of this illustrative example the desired intentions $w_1^* = 10867.5$ and $w_4^* = 126880.5$ appeared to be overattainable. By virtue of theory stated in [17] it means that it is possible to improve the model (6), (7) (in the sense of Pareto). This can be achieved by applying one of the modifications of goal attainment method of Gembicki, for example, one of the approaches developed in [29] and [30].

V. CONCLUSIONS

In the present paper, one agrarian problem is formulated for finding a guaranteed harvest of cereal crops, the yield of which depends on external factors, in particular, on soil-climatic conditions. For the formulated agrarian problem, a mathematical model is constructed, which is a multi-criteria problem with constraints. Further, the essence of the three main methods for solving the constructed mathematical model is briefly described: the weighted sum approach method, the epsilon-constraint method and the goal attainment method of Gembicki. The main disadvantages of the weighted sum approach and epsilon-constraint method are briefly analyzed and associated with the ambiguity in the selection of criteria's importance coefficients. Then, the goal attainment method of Gembicki is applied to the constructed mathematical model. Besides, in this paper, a computational example is formulated, and its mathematical model is constructed and solved by the goal attainment method of Gembicki. Finally, the authors would like to point out that the agricultural problem considered in this paper, the detailed course of its modelling as well as more or less detailed description of at least one approach to its solving (goal attainment method of Gembicki), in our opinion, may turn out to be a useful template for a wide range of users (not necessarily well-knowing higher mathematics) related to management of growing and production of cereal crops under various possible external factors, in particular, under various soil-climatic factors (which are with increasing frequency occurring even in such a small country as Latvia), under various diseases of cereal crops: infectious diseases (viral and fungal diseases) caused by macro- or micro-organisms; non-infectious diseases caused by inorganic nature, etc. It is important to note that the algorithm proposed in this paper for solving the constructed mathematical model is easily implemented both in Microsoft Excel software and in Mathcad, MATLAB etc. softwares (Mathcad is designed for the engineering environment, while MATLAB betrays its roots as a mathematics tool designed for mathematicians clearly).

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