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Subjective Probabilities Elicitation and Combination in Risk Assessments Problems

Oleg Uzhga-Rebrov
Rezekne Academy of Technologies
Institute of Engineering
Rezekne, Latvia
rebrovs@tvnet.lv

Peter Grabusts
Rezekne Academy of Technologies
Faculty of Engineering
Rezekne, Latvia
peteris.grabusts@rta.lv

***Abstract*. Very many areas of human activity are associated with greater or lesser risks. In order to make reasonable decisions, these risks must be properly assessed. The consequences of any risk can be characterized on the basis of two fundamental dimensions (metrics): (1) losses associated with the outcomes of implementation an unfavourable event; (2) probabilities that quantify the uncertainties in the occurrence of these outcomes. This article in a concise form presents and analyses approaches to subjective probabilities elicitation and combining the obtained individual estimates in group subjective probabilities estimation.**

Keywords: risks, subjective probabilities elicitation, subjective probabilistic estimates combination, unfavourable event's outcomes.

I. INTRODUCTION

Almost all people are well aware that any risks are associated with some unfavourable circumstances (events). Such events can be various kinds of natural disasters, fires, vehicle accidents, financial losses, injuries, epidemics, and many others.

Usually, any individual on a subconscious, intuitive level definitely understands what risk is. But if you ask him to give a formal definition of risk, he will definitely be at a loss to give a meaningful definition of this concept. As figuratively noted in [1], if you ask ten different people what they mean by the word "risk", you will probably get ten different answers.

A large number of formal definitions of risk have been proposed. Sufficiently broad summaries of such definitions are presented in [2], [3]. As an example, we present a summary of definitions from [3] without references to original sources.

1. Risk is an estimate of the probabilities and weights of unfavourable consequences.
2. Risk is a set of triplets (s_i, p_i, c_i), where s_i - i-th scenario, p_i - likelihood of the scenario, s_i, c_i - consequences of the scenario s_i, $i = 1,...,N$.
3. Risk is equal to the product of probability and damage.
4. Risk is a situation or event where human values (including people themselves) are accepted as bets where the outcomes are uncertain.
5. Risk is a combination of primitive concepts: outcome, likelihood, importance, causal scenario, and identified population.
6. Risk is an expression of the influence and possibility of an accident in terms of the severity of the potential accident and the likelihood developments.
7. Risk is a combination of probabilities and limits of consequences.
8. Risk is the uncertainty of an event or activity associated with some human values.
9. Risk is equal to the expected damage.
10. Risk is the likelihood of injury, illness or harm to health because of the dangers.
11. Risk is the effect of uncertainty on goals.

In this work, the definition presented in [2] is taken as a basis. "*Risk refers to uncertainties about the extent to*

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which events and their consequences (or their outcomes) affect what constitutes human value".

At a lower level, risk can be given an operational definition [1]. Such a definition specifies the concepts of factors included in the abstract definition of risk.

The instrumental level of definition contains the expression of risk in terms of one or more risk metrics. The term *risk metric* is used interchangeably with *risk assessment* and is defined as "a mathematical function of probability of an event and the consequences of that event".

Reference [4] provides conceptual definitions of risks. The main procedures (risk assessment and management) are presented in ISO 31000 [4], ISO 31010 [5].

The following concepts and definitions are taken from ISO 31010 [5].

- *Risk assignment* is the overall process of risk identification, risk analysis and risk assessment.
- *Risk analysis* is the process of recognizing the nature of risk and determining the level of risk.
- *Risk assessment* is the process of comparing the results of a risk analysis with risk criteria to determine whether the risk and/or its magnitude is supportable or tolerable.

Two factors (dimensions) are used to characterize any risk: (1) losses associated with the outcomes of an unfavourable event; (2) estimates of uncertainties in the occurrence of outcomes.

Subjective estimation of probabilities is widely used in situations where, due to the lack of statistical data, an objective estimation of these probabilities is impossible. Examples of such situations might be:

- Assignment of risks when making decisions on the strategic development of a business (Choosing the country where a new enterprise is located, making a decision on the release of new products, making a decision on investing in risky activities).
- Assigning risks to political decisions regarding relations with partner countries and hostile countries.
- Appointment of risks when planning decisions during military conflicts.
- Assessment of environmental risks (risks of negative impacts on the environment, risks of negative impacts of the external environment on infrastructure facilities and searches for negative impacts of invasive plant and animal populations).

The purpose of this paper is to present the most common methods for the subjective evaluation of deterministic values of probabilities in risk assignment problems.

II. SUBJECTIVE PROBABILISTIC ESTIMATES ELICITATION

In the subjective assessment of probabilities in the problems of risk assessment, they deal with fundamentally different types of uncertainties.

- *Aleatory uncertainty*, which refers to the chances of occurring the outcomes of unfavourable events. This uncertainty is an inherent property of the phenomena of the world around us. The task of the expert (experts) is to quantify the relevant aleatory uncertainties using probabilistic estimates.

- *Epistemic uncertainty* characterize the degree of confidence of the expert (experts) regarding the estimated probabilities values. When performing expert estimates of probabilities, all possible conditions must be provided to reduce epistemic uncertainties.

The processes of expert estimation of probabilities are presented in details in industry guidelines [7 - 10]. Detailed information on probabilities elicitation can also be found in [11].

In general, approaches to probability-based characterization of uncertainties can be divided into five big categories [8]:

- frequency;

- based on judgments / subjective;

- scenario analysis;

- others (interval probabilities, fuzzy logic, meta-analysis);

- methods of sensitivity analysis.

The approaches considered in this paper explicitly belong to the 2nd category.

Let us present widely used approaches to elicitation of subjective probabilities.

1. *Direct estimation of relevant probabilities*. Alternatively, this method is called the *fixed probability method.*

Using knowledge and experience, the expert directly assigns the required probabilistic estimates. It can express the uncertainty of its judgments with the help of fractiles and extreme values for the estimated point values (see Fig. 1).

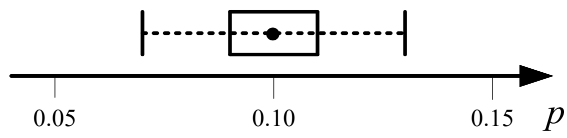

Fig. 1. Graphical representation of the uncertainty of expert with respect to the actual value of the estimated probability.

In this figure, the dot represents the median (average) assessment of expert. The values $p = 0.09$ and $p = 0.11$ correspond to fractiles 0.25 and 0.75. The values $p = 0.07$ and $p = 0.13$ are the extreme possible values of the estimated probability.

2. *Preliminary verbal estimation of probabilities.*

This approach is based on the fact that verbal categories of values of the estimated probabilities are determined first [12]. As an example, Table 1 presents a possible option for determining verbal categories on a set of probabilistic estimates.

TABLE 1 VERBAL VALUES OF PROBABILITIES AND CORRESPONDING INTERVALS OF PERCENTAGE VALUES OF THESE PROBABILITIES

Verbal categories	Percentage values of probabilities
In higher degree no chance	01 – 05%
Very implausible	05 – 20%
Implausible	20 – 45%
About the same chances	45 – 55%
Plausible	55 – 80%
Very plausible	80 – 95%
In higher degree plausible	95 – 99%

Verbal assessments of probabilities of outcomes and losses associated with these outcomes are used in formalization of risk matrices to categorize existing risks. However, in many risk assessment problems, the probabilities of all outcomes can have rather small values. Therefore, the use of verbal categories similar to those presented in Table 1 seems inappropriate. In such cases, it may be recommended to develop a specific system of verbal categories corresponding to the possible numerical values of the probabilities in a particular task of risk assessment. Using such a system, an expert can first set verbal categories for the estimated probabilities and, on this basis, subsequently proceed to numerical estimates of the required probabilities.

3. *Use of reference lotteries.*

The idea of this approach is to use specific lotteries, on the basis of which the probabilities of interest can be estimated. First, a lottery is set, the outcomes of which are understandable and acceptable to the expert. One of the lottery outcomes must be clearly preferable to the other. For example, consider a lottery with the following outcomes:

Outcome 1 - getting a new luxury car if outcome A comes true.

Outcome 2 - free lunch at a restaurant if outcome A does not come true.

The outcomes of the second lottery are the same, but they are related to the probabilities of the implementation and non-implementation of outcome A. This lottery is called the *reference lottery*. Both lotteries are presented in Fig. 2 in the form of a decision tree.

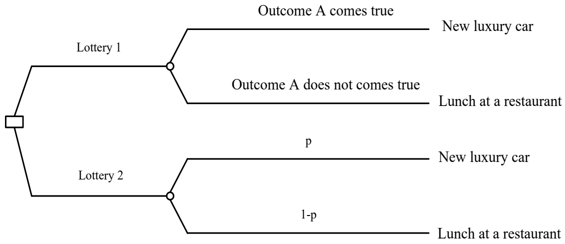

Fig. 2. Decision tree for estimating the probability of outcome A by comparing lotteries.

The expert is asked to determine such a value of probability, at which he will be indifferent in which of the lotteries to take part.

4. *Pairwise comparison of the plausibility of outcomes.*

With a large number of outcomes of an unfavourable event, assigning their probabilities becomes difficult for an expert. In [13], it is proposed to use the approach - the analytic hierarchy process (AHP) [14], with the help of which the process of subjective estimation of probabilities is reduced to a paired comparison of outcomes according to the degree of plausibility of their implementation. Additional information about AHP can also be found in [15].

Let an unfavourable event be associated with outcomes $o_1,..,o_n$. These outcomes are presented in the 1st row and 1st column of Table 2. The value $s_{ij}, i, j = 1,...,n$, in the corresponding cell of the table represents an expert assessment of the superiority in chances of the outcome o_i over the chances of the outcome o_j. Evaluation is made on a 9-point scale Saaty [14]. Note, that the diagonal cells of the table contain 1 and $s_{ji} = 1/s_{ij}$.

TABLE 2 RESULTS OF PAIRWISE COMPARISON OF OUTCOMES IN TERMS OF THE DEGREE OF PLAUSIBILITY OF THEIR IMPLEMENTATION

	o_1	o_2	$\cdots$	o_{n-1}	o_n
o_1	1	s_{12}	$\cdots$	$s_{1,n-1}$	s_{1n}
o_2	s_{21}	1	$\cdots$	$s_{2,n-1}$	s_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
o_{n-1}	$s_{n-1,1}$	$s_{n-1,2}$	$\cdots$	1	$s_{n-1,n}$
o_{n1}	s_{n1}	s_{n2}	$\cdots$	$s_{n,n-1}$	s_{nn}
	$\sum_{i=1}^{n} s_{i1}$	$\sum_{i=1}^{n} s_{i2}$	$\cdots$	$\sum_{i=1}^{n} s_{i,n-1}$	$\sum_{i=1}^{n} s_{in}$

After filling the table, the sums of values $\sum_{i=1}^{n} s_{ij}$ in its columns are calculated and the original values s_{ij} are normalized relative to these sums. Another table is compiled containing the calculated normalized values s_{ij}^n.

Next, the normalized eigenvector of the pairwise comparison table is determined $W=\{w_i / i=1,...,n\}$. i -th component of vector w_i is calculated by the expression

$$w_i=\sum_{j=1}^{n} s_{ji} / n \ , \ i=1,...,n \ , \quad (1)$$

where n - the number of outcomes.

The values w_i are taken as estimated values of the probabilities of outcomes $p(o_i)$.

To assess the degree of consistency of the expert's subjective assessments, the following sequence of calculation procedures is performed.

1. The *main eigenvalue* of the table is calculated

$$\lambda_{\max}=\sum_{i=1}^{n}\frac{w_i}{\sum_{i=1}^{n} s_{ij}}, \ j=1,...,n, \quad (2)$$

2. The *index of consistency* of pairwise comparisons is calculated by the expression

$$CI=\frac{\lambda_{\max}-n}{n}. \quad (3)$$

3. From Table 3, the value of *random consistency index* RI is determined. These values are a function of the number of pairwise comparisons n .
4. The value of the *consistency ratio* CR is calculated

$$CR=\frac{CI}{RI} \ . \quad (4)$$

TABLE 3 RANDOM CONSISTENCY INDEX VALUES RI

n	RI
1	0
2	0
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.49

With values $CR<0.10$, the results of pairwise comparisons are considered to be reasonably consistent. With values $CR>0.10$, the expert may be asked to clarify all or some of the results of his pairwise comparisons.

III. COMBINATION OF SUBJECTIVE PROBABILITIES

If probabilities elicitation is performed by a group of experts, the problem of combining individual expert assessments appears. Let us present some formal methods for solving this problem based on the data presented in [16].

1. *Linear opinion pool.*

Let expert estimation of the probability of some outcome o_j be performed by n experts. The combined estimate of this probability can be calculated by the expression

$$p(o_j)=\sum_{i=1}^{n} w_i p_{ij} \quad (5)$$

where p_{ij} - evaluation of the probability of the outcome o_j by the i -th expert; w_i , $i=1,...,n$, - weight given to the assessment of the i -th expert.

The value $p(o_j)$ is a weighted linear combination of the assessments p_{ij} . Weight w_i characterizes the degree of confidence in the assessment of the i -th expert.

The combined assessment $p(o_j)$ satisfies the unanimity property: if all experts agree with their own assessments, they must agree with the combined assessment.

2. *Logarithmic opinion pool.*

The combined value $p(o_j)$ can be calculated by the expression

$$p(o_j)=k\prod_{i=1}^{n}(p_{ij})^{w_i} \ , \quad (6)$$

where k - normalizing constant.

As noted in [16], expression (6) satisfies the external Bayesian principle. Let us assume that the combined value of the probability $p(o_j)$ is calculated from expression (6) and new information is obtained, on the basis of which the values p_{ij} and $p(o_j)$ can be re-evaluated. There are two alternative ways to perform such a re-evaluation: (1) first, using new information, experts re-evaluate their initial estimates p_{ij} , then the new estimates are combined according to expression (6); (2) using the new information, the resulting value $p(o_j)$ is re-evaluated. According to the external Bayesian principle, the results should match in both cases.

3. *Bayesian combination of probabilities.*

In [16], four variants of such a combination are presented. Let's present one of the most used options. Let p_{ij} , $i=1,...,n$ be the probability estimated by the i -th expert that the outcome o_j will come true. Expressed in terms of posterior odds, the credibility of the outcome $q^*=\frac{p^*}{1-p^*}$ occurring is calculated by the expression

$$q^*=\frac{p_0}{1-p_0}=\prod_{i=1}^{n}\frac{f_{1i}(p_i / q=1)}{f_{0i}(p_i / q=0)} \ , \quad (7)$$

where f_{1i} - assessment of the probability by the i -th expert under the condition of the implementation of the outcome

o_j; f_{0i} - assessment of the probability by the i-th expert under the condition of non-implementation of the outcome o_j.

There are many other methods for combining subjective probabilistic estimates. Due to space limitations of the article, these methods are not considered here.

Let us consider the possibility of combining subjective probabilistic estimates with possible objective values of these probabilities. Note, that the objective values of probabilities are not reliable enough to be used as basic estimates.

The idea of such a combination of probabilistic estimates obtained from various sources is presented in [17]. Let there be a complete system consisting of two random events. In the context of risk assessment, the first random event is the implementation of an outcome o_j with probability $p(o_j)$; the second random event is the non-implementation of the outcome o_j with probability $1-p(o_j)$. Uncertainty about a value $p(o_j)$ can be modelled using β distribution. In the general case, the density function of β distribution of a random variable X is represented by the expression

$$f(X)=\frac{(a+b+1)!x^a(1-x)^b}{a!b!}, \quad (8)$$

The expected value of the random variable X is

$$E(X)=\frac{a+1}{a+b+2}. \quad (9)$$

Let the expert subjectively assessed the probability of implementation of the outcome O_j as $p(o_j)=0.10$. Then, considering this probability as its expected value, we have

$$0.10=\frac{a+1}{a+b+2}. \quad (10)$$

The sum $(a+b)$ can be interpreted as the hypothetical number of attempts that were used to estimate the value of $p(o_j)$. Then a is interpreted as a hypothetical number of realizations of the outcome o_j in $(a+b)$ attempts. By assigning a hypothetical number of attempts $(a+b)$, the expert can express his degree of confidence in the value of $p(o_j)$. If the expert is not very confident in his estimate, a value of up to 10 can be assigned to $(a+b)$.

Let the expert subjectively estimate $a+b=8$, $a=1$. Then $b=7$.

Let us now assume that a computer simulation of the outcome o_j implementation process has been performed. As a result, it was obtained that the outcome o_j was realized once in 10 attempts.

Assuming that the conjugate posterior distribution for $p(o_j)$ is β distribution, the posterior value of probability $p^*(o_j)$ can be calculated by the expression

$$p^*(o_j)=\frac{m+a+1}{n+a+b+2}. \quad (11)$$

In our case, $m=1$, $n=10$. Substituting these values into expression (11), we have

$$p^*(o_j)=\frac{1+1+1}{10+1+7+2}=\frac{3}{20}=0.15. \quad (12)$$

This posterior value of the probability $p^*(o_j)=0.15$ seems to be more reliable than its prior value $p(o_j)=0.10$.

IV. CONCLUSIONS

Currently, expert estimates of probabilities are very widely used in various areas of human activity, including risk assessment. The prevalence of such assessments is evidenced by the fact that they are used in more than 80% of stochastic models in making strategic and tactical business decisions.

The importance of subjective assessments of probabilities in problems of risk assessment is evidenced by the fact that the costs in problems of subjective estimation in complex multidimensional studies range from $200,000 to $2 million [7].

In recent decades, effective approaches to the subjective estimation of probabilities have been proposed. Education and special training of experts is important for qualitative assessments. This makes it possible to obtain sufficiently reliable results even in very complex tasks of risk assessment.

This article is mainly of a review nature. It presents and analyses in a concise form the main elicitation method of subjective probabilistic estimates and methods for combining such estimates.

Directions for further research in this area are the analysis of existing approaches to evaluating probabilities using interval and fuzzy estimates in order to use them in specific risk assignment problems.

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