

Study of the Kinematic and Dynamic of Interaction of a Vibrating System for Cutting Optical Slugs

Angel Lengerov

Department of Mechanical
Engineering and Technologies
Technical University of Sofia,
Plovdiv Branch
Plovdiv, Bulgaria
email anlen@tu-plovdiv.bg

Kalin Krumov

Department of Mechanical
Engineering Equipment and
Technologies
Technical University of Gabrovo,
Gabrovo, Bulgaria
email
kalin_krasimirov_krumov@abv.bg

Georgi Levicharov

Department of Mechanical
Engineering and Technologies
Technical University of Sofia,
Plovdiv Branch
Plovdiv, Bulgaria
email glevi@tu-plovdiv.bg

Abstract. In the article, studies of the kinematics and dynamics of interaction of the elements of the vibration system "slug-cut disk" are made. A mathematical model describing the trajectory of a cutting arm has been created and the technological parameters of the system at which the phenomenon of resonance occurs have been determined.

Keywords: kinematic, dynamic, study, vibrating system, optical slug, cutting.

I. INTRODUCTION

Mechanical systems, in which the processing is realized by introducing systematic contacts between two objects, are called vibro-impact systems [5,6].

Vibro-impact modes of processing are the basis of a different kind of actions of a wide spectrum of machines, tools and devices of different functional type [2,3,4,7,8,9]. Characteristic of these systems are the periodic modes of motion, carried out according to the scheme of forced oscillations or self-oscillations.

The theoretical evaluation of the degree of influence of the caused under conditions of vibro-impact cutting forced oscillations of optical slugs can be represented by mathematical models of the kinematics and dynamics of the interaction between a diamond cutting disk and an optical workpiece.

II. MATHEMATICAL MODEL

The mathematical description of the trajectory and the laws of movement of the cutting arm with a centrifugal vibrator, makes it possible to analytically determine the efforts of the contact interaction between the cutting tool

and the machined workpiece at preset magnitudes of the forced oscillations.

The developed mathematical model allows for studying the kinematics and dynamics of the interaction between the elements of the vibrational system "workpiece – cutting disk" (Fig.1).

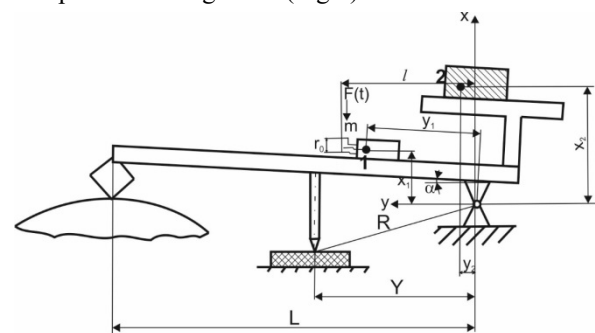


Fig. 1. Mathematical model of the cutting system.

In order to simplify the calculations, the action of the centrifugal vibrator is replaced by a driving harmonic force $F(t)$. According to [1] this is permissible, since the amplitude of the caused oscillations is much smaller than the radius of motion of the imbalance. The cutting arm in the hinged device is considered to be fixed so, that oscillations in the direction of the axis and perpendicular to the plane of the drawing do not exist. To simplify the calculation, the cutting arm is regarded as an absolutely rigid body, and the masses of the cutting arm, the vibrator and the devices, creating a static load on the workpiece, are replaced by two concentrated masses (points 1 and 2

Print ISSN 1691-5402

Online ISSN 2256-070X

<https://doi.org/10.17770/etr2023vol3.7242>

© 2023 Angel Lengerov, Kalin Krumov, Georgi Levicharov. Published by Rezekne Academy of Technologies.
This is an open access article under the [Creative Commons Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/).

in Fig.1). The axis of rotation of the cutting arm is taken as the origin of the coordinate system. The notations, used in the mathematical model, are as follows:

α - an angle, determining the position of the rocker, i.e., an angle between the origin and the direction of the axis of rotation - the point of contact "optical slug - cutting disk" in the counterclockwise direction;

m - mass of the eccentric;

m_1 - mass of the system (without the counterweight);

m_2 - counterweight mass;

k_1 - damper elasticity;

k_2 - elastic support "detail-cutting disc";

ξ - effective damping coefficient.

2.1. Study of the kinematics

The mass moment of inertia I of the system about the axis of rotation can be represented as

$$I = m_1(x_1^2 + y_1^2) + m_2(x_2^2 + y_2^2). \quad (1)$$

The angle α_e between the contact point of the support with the damper, and the contact point of the optical slug with the cutting disk, is calculated by the dependence

$$\alpha_e = -\frac{\Delta h}{L}, \quad (2)$$

where Δh is the distance between the optical workpiece and the cutting disk at the moment of contact of the support with the damper.

The equilibrium equation of the system will take the form

$$(m_1 y_1 + m_2 y_2)g = \left(\alpha_0 + \frac{\Delta h}{L} \right) k_1 y^2 + \alpha_0 k_2 L^2, \quad (3)$$

where α_0 is the angle of the state of equilibrium.

Consequently,

$$\alpha_0 = \frac{(m_1 y_1 + m_2 y_2)g - \frac{\Delta h k_1 R y}{L}}{k_1 y^2 + k_2 L^2}. \quad (4)$$

The driving harmonic force is presented in the following form

$$F(t) = m r_0 (2\pi\nu)^2 \sin(2\pi\nu t) \quad (5)$$

Then for the driving moment we obtain

$$Me(t, \nu) = m r_0 l (2\pi\nu)^2 \sin(2\pi\nu t) \quad (6)$$

The resultant moment of the acting forces (the weight and the reactions of the supports), depending on the quantity $\delta = \alpha - \alpha_0$, i.e., the angular deviation of the arm position from the equilibrium state, is determined as it follows

$$\hat{M}(\delta) = \begin{cases} -\delta(k_1 y^2 + k_2 L^2) & \text{npu } \delta \geq -\alpha_0 \\ \alpha_0 k_2 L^2 - \sigma k_1 y^2 & \text{npu } \delta < -\alpha_0 \end{cases}. \quad (7)$$

The motion equation of the system can be represented as

$$I \ddot{\delta} = \hat{M}(\delta) - \xi \dot{\delta} + Me(t, \nu). \quad (8)$$

After replacing the variables

$$Y_1 = \dot{\delta} \quad Y_0 = \delta \quad \dot{Y}_0 = Y_1. \quad (9)$$

Then equation (8), taking into account (9), takes the form

$$\dot{Y}_1 = \frac{1}{I} \left[\hat{M}(Y_0) - \xi Y_1 + Me(t, \nu) \right]. \quad (10)$$

For small disturbing forces (in case of small masses, at which the angle of deviation from the equilibrium position does not exceed α_0) there is an analytical solution.

In this case equation (8) can be written in the form

$$I \ddot{\delta} + \xi \dot{\delta} + (k_1 Y^2 + k_2 L^2) \delta = Me(t, \nu), \quad (11)$$

where it is convenient to present $Me(t, \nu)$ as

$$Me(t, \nu) = m r_0 l (2\pi\nu)^2 \exp^{i2\pi\nu t}. \quad (12)$$

Under a steady mode of operation of the system, the solution has the form

$$\delta(t) = A \exp^{i2\pi\nu t}. \quad (13)$$

Substituting (13) into (11) with consideration of (12), we obtain

$$\begin{aligned} -A(2\pi\nu)^2 I + i2\pi\nu \xi A + (k_1 Y^2 + k_2 L^2) A &= \\ = m r_0 l (2\pi\nu)^2. \end{aligned} \quad (14)$$

Hence,

$$A = \frac{mr_0 l (2\pi\nu)^2}{(k_1 Y^2 + k_2 L^2 - (2\pi\nu)^2 I + 2\pi\nu \xi i)} \quad (15)$$

From (15) we find

$$|A| = \frac{mr_0 l (2\pi\nu)^2}{\sqrt{(k_1 Y^2 + k_2 L^2 - ((2\pi\nu)^2 I)^2 + (2\pi\nu \xi)^2)}} \quad (16)$$

The resonant frequency will be

$$\nu_0 = \frac{1}{2\pi} \sqrt{\frac{k_1 Y^2 + k_2 L^2}{I}} \quad (17)$$

In cases of large disturbing forces, when the angle of deviation of the cutting arm relative to its equilibrium position exceeds α_0 , i.e., when a vibro-impact mode of cutting is realized, the solution of the equation of motion of the system (8), expressed numerically, makes it possible to theoretically obtain the trajectories of motion of the cutting arm (Fig. 2).

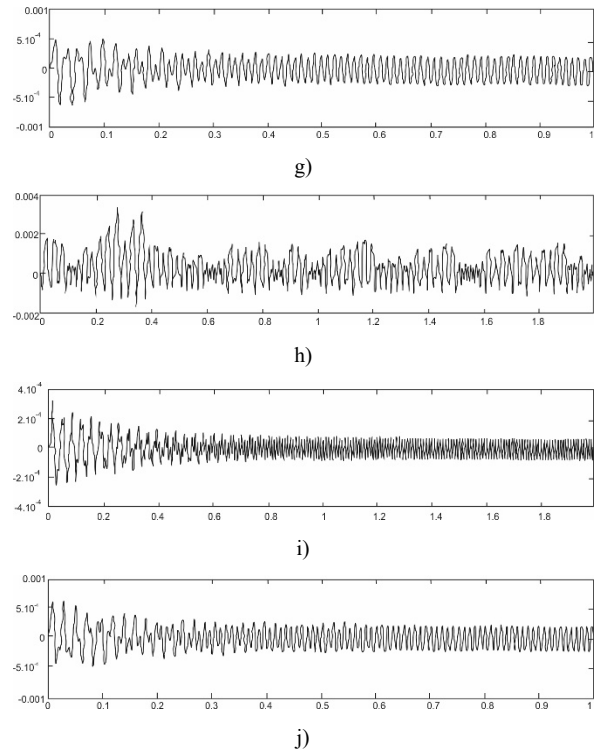
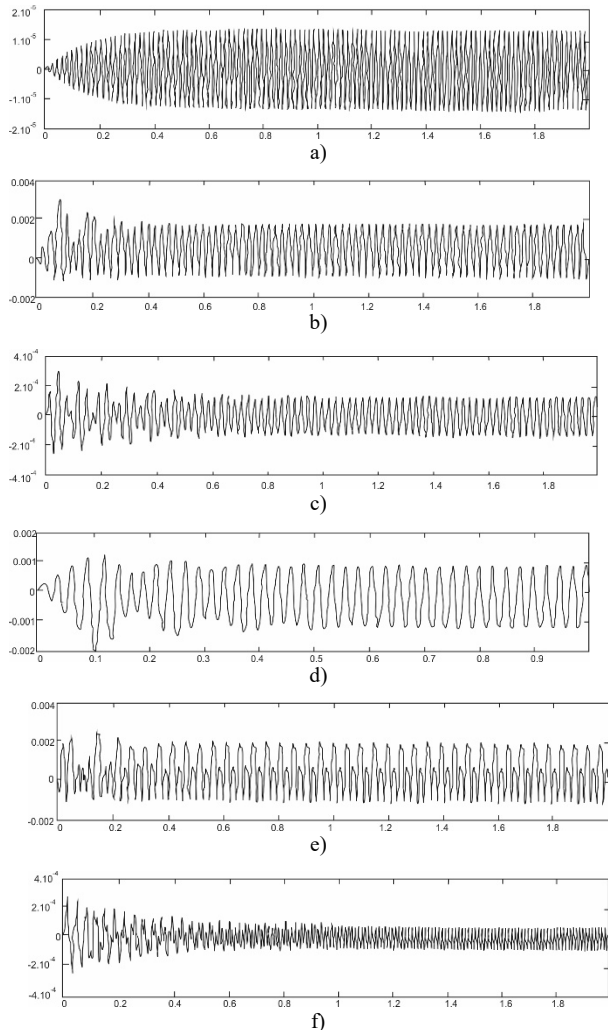


Fig. 2. Amplitude-frequency spectrum of the oscillating movements of the cutting arm: a) for a mode of processing with no impact at theoretical resonant frequency $\nu_0 = 57 \text{ Hz}$; b), c), d) $\nu_0 = 41 \text{ Hz}$; e), f), g) $\nu_0 = 85 \text{ Hz}$; h), i), j) $\nu_0 = 101 \text{ Hz}$; b), e), h) displacement $y_1 = 0,17 \text{ m}$; b), c), f) i) displacement $y_1 = 0,37 \text{ m}$; b), e), h) mass $m_1 = 0,45 \text{ kg}$; d), g), j) mass $m_1 = 1,45 \text{ kg}$.

It is observed, that the frequency of the forced oscillations, for which the trajectory of movement was obtained in the cases of separation of the workpiece from the cutting disk (Fig.2 b, e, h), corresponds to the frequency of the vibro-impact modes 1-3, used in the tests for cutting optical materials.

The movement of the vibrator along the cutting arm in the direction of the machined workpiece (Fig. 2 c, f, i) leads to a significant decrease in the amplitude of oscillations (up to 10 times) which, in turn, reduces the ratio between the distances l/L , where l/L are respectively the distances from the axis of rotation of the cutting arm to the points of attachment of the vibrator and the workpiece. The reduction of the disturbing forces on the cutting arm is well presented in Fig. 2 i. When placing the vibrator close to the axis of rotation of the cutting arm and at its maximum speed of rotation (Fig.2h), stable nature of oscillations is observed, with high frequencies and an amplitude, one order of magnitude lower.

The increase in the mass of the system (e.g., when using a larger size electric motor) will increase its inertia, which leads to a decrease in the frequency of oscillation of the cutting arm and decrease in its amplitude (Fig.2 d, g, j).

2.2. Study of the dynamics

To evaluate the influence of the vibro-impact mode of processing when cutting optical slugs on the intensity of the process, it is necessary to study the forces, acting on the side surfaces of the cutting disk and the workpiece.

Fig. 3 illustrates part of the trajectory of movement of the cutting arm for the time of contact between the workpiece and the cutting disk. In order to perform a comparative analysis of the movements in the system, the magnitude of the angle of equilibrium α_0 was taken into account. This angle takes into account the deviation of the cutting arm from its initial position when the cutting tool comes into contact with the optical workpiece. The ratio between the forces, acting in a standard cutting process, and in case of forced oscillations toward the machined workpiece, will be proportional to the ratio of the perpendiculars to the straight line AE, drawn from the points B and G, respectively. It can be seen from Fig. 3 that the magnitudes of the forces of interaction between the sample and the tool in a vibro-impact mode of cutting will be significantly higher, which leads to an increase in the depth of the pre-destructive layer in the material of the workpiece, and an increase in the processing intensity.

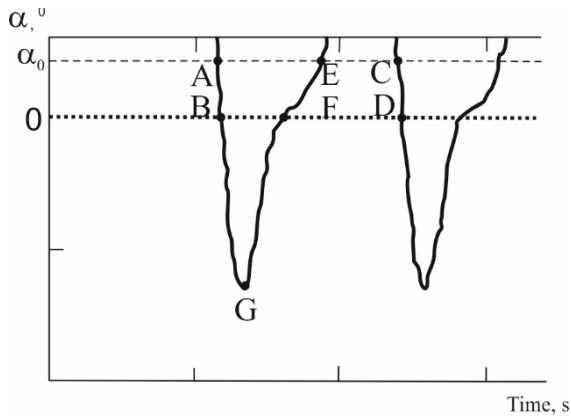


Fig. 3. Trajectory of movement of the cutting arm with the machined workpiece in the contact zone with the cutting disk.

Taking into account the fact, that the constituent forces intersect with the constituents of the free oscillations in the system, we can consider that the phenomenon of nonlinear resonance occurs.

Then the law of motion of the vibro-impact system takes the form

$$x(t) = A \cos(\omega t + \varphi) - Jx(t) \quad , \quad (18)$$

where A is the amplitude, determined when there is no impact motion;

$$J = \int_{t_\alpha}^{t_\alpha + \Delta} \Phi(x, \dot{x}) dt \quad - \text{impulse for the contact time } \Delta \quad (19)$$

$$\Phi(x, \dot{x}) \quad - \text{force of the impact interaction;}$$

$x(t)$ - the resulting reaction in the system of the force interaction.

To determine the unknown quantities J and φ , we set the conditions

$$x(0) = x_0, J(\alpha) = m(1+K)x_0. \quad (20)$$

Then $x_0(t) = A \cos \varphi - Jx(0)$,

$$J = \left[-\omega A \sin \varphi - J \dot{x}(0) \right] (1+K)m.$$

The evaluation of the stability of the oscillations, described by (18), and fulfilling the condition $x \leq x_0, J \geq 0$, is performed after setting the parameters of the system based on the energy function

$$E(J, \varphi) = E_1(J, \varphi) - E_2(J, \varphi),$$

where $E_1(J, \varphi)$ is the work of the forces, created by an outer source;

$E_2(J, \varphi)$ - the work of the dissipation forces.

According to [10], the inequality

$$\left. \frac{dE}{dJ} \right|_{\substack{J=J_0 \\ \varphi=\varphi_0}} = \left(\frac{\partial E}{\partial J} + \frac{\partial E}{\partial \varphi} \frac{\partial \varphi}{\partial J} \right) \bigg|_{\substack{J=J_0 \\ \varphi=\varphi_0}} > 0, \quad (21)$$

is a sufficient condition for the instability of the considered system at $J = J_0, \varphi = \varphi_0$.

The most intense resonant modes and those, which are close to the free modes, significantly differ from the natural frequency of the system. This makes the first term in the right-hand side of the dependence (18) much smaller than the second.

With this in mind, we find the approximate solution for the resonant vibro-impact modes

$$x(t) = -Jx(t). \quad (22)$$

At $x(0) = x_0$ we have $J = -\frac{x_0}{x(0)}$ and

$$x(t) = \frac{\Delta}{x(0)} x(t).$$

At small amounts of damping due to the external influence, the expression (21) uniquely describes the resonant modes of the system.

Putting $P(t) = P \cos(\omega t + \varphi)$ we will find the conditions for the existence of the quasi-resonant movements at any multiple of l , while balancing the work of friction forces and disturbance forces.

The period of these oscillations $T = 2\pi l / \omega$.

The energy balance equation describing the condition for maintaining the resonance mode takes the form

$$E_1 = E_2 + E_3, \quad (23)$$

where $E_1 = \int_0^{\tau_1} P(t) \dot{x}(t) dt$ is the work done by external sources;

$$E_2 = \int_0^{\tau_1} G^{-1} [\varphi_\alpha x(t)] \dot{x}(t) dt \quad \text{- the loss of energy;}$$

$$E_3 = J^2 \frac{1-K}{2m(1+K)} \quad \text{- the work of the forces in case of contact friction.}$$

From the dependence of the magnitude of the impact pulses on the frequency of the external impact, the unstable modes of the system can be determined.

At higher levels of damping and excitation, the oscillations will not have a resonant character. For a system with $x_0 > 0$, with a gradual increase in the frequency of external impacts, the highest intensity vibro-impact modes can be realized, in which, when reaching certain characteristic magnitudes of the frequencies, determined by the conditions of the energy balance, the forces of excitation and damping decay.

For a system with $x_0 < 0$, the opposite situation occurs, and the oscillations in such a system arise after the transmission of additional initial impulses. When $x_0 = 0$, the most intense oscillations are possible to occur over the full range of used frequencies. As the natural frequencies increase, the influence of the dissipation factors also increases and the limiting magnitudes of the impact pulses decrease.

III. CONCLUSION

On the basis of the proposed mathematical model for studying the kinematic and dynamic conditions of interaction between the machined workpiece and the cutting disk, trajectories of the oscillation displacements of the workpiece relative to the cutting tool have been obtained both for continuous (stationary) and vibro-impact modes of operation when cutting optical workpieces.

It has been established that the used modes of operation of vibrators with frequencies, equal to or multiple of the resonant frequency of the cutting arm, allow to obtain stable amplitudes of oscillations.

The change in the position of the vibrator of the cutting arm to a side of the machined workpiece, as well as the increase in its mass, lead to an increase in the static

load on the workpiece and causes a decrease in the amplitude oscillations, which, in turn, reduces the processing performance.

IV. ACKNOWLEDGMENTS

This work was supported by the European Regional Development Fund within the OP "Science and Education for Smart Growth 2014-2020", Project Competence Centre "Smart Mechatronic, Eco-And Energy Saving Systems And Technologies", № BG05M2OP001-1.002-0023.

REFERENCES

- [1] I.M. Bykovsky Fundamentals of the theory of vibration technology. M.: Mashinostroenie, 1998, p.362.
- [2] Sergiev A P, Vladimirov A A, Makarov A V and Shvachkin E G, The physical basis of the process of vibration cutting during turning, Scientific-theoretical journal Herald of the Belgorod State Technological University 3, pp.94 – 102, 2017.
- [3] Sergiev A P, Vladimirov A A and Shvachkin E G, Optimization of technological parameters during vibration cutting, Vestnik mashinostroeniya 3, pp. 79 – 82, 2017.
- [4] Mikheyev V V, Saveliev S V and Permyakov V B Comprehensive approach to the selection of the optimal energy-effective mode of operation of vibration rollers, Problems of mechanical engineering. Materials of the 3rd International Scientific Conference pp 158-165, 2019.
- [5] S. Sabev, P. Kasabov. The effect of feed rate and cutting speed to surface roughness during hole boring of aluminum with anti-vibration boring bar. Environment. Technologies. Resourced. Proceeding of the International Scientific and Practical Conference, 2021. 3. 314-318, doi: 10.17770/etr2021vol3.6537.
- [6] S. Sabev, P. Kasabov. The influence of feed rate and cutting speed to surface roughness during hole boring of AISI 304 with anti-vibration boring bar. AIP Conference Proceedings. 2449.060004.10.1063/5.0091006.
- [7] Śniegulska-Grądzka D, Nejman M and Jemielniak K, Cutting force coefficients determination using vibratory cutting, Ischia: 10th CIRP Conference on Intelligent Computation in Manufacturing Engineering – CIRP ICME '16. Procedia CIRP 62, pp. 205 – 208, <https://doi.org/10.1016/j.procir.2016.06.091>.
- [8] Wang G F, Hu Y B, Zhu W Q, Bao Z Y, Chen Z J, Huang Z H and Wang W, The design of a compaction parameters management system for intelligent vibratory roller, IEEE 3rd Information Technology and Mechatronics Engineering Conference (ITOEC) pp 634 – 638, 2017, DOI: 10.1109/ITOEC.2017.8122375.
- [9] Hyung Tae Kim, An Mok Jeong, Hyo Young Kim, Jong Wook An, Cheol Ho Kim, Kyung Chan Jin and Seung-Bok Choi, Lateral vibration control of a precise machine using magneto-rheological mounts featuring multiple directional damping effect, Smart Materials and Structures, Volume 27, Number 3, 2018, DOI 10.1088/1361-665X/aaad9e.
- [10] G. Scott. Experimental estimation of thermal processing possibilities of crystal glass. New works on friction and wear. M., 1989, pp. 78-84.