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Longitudinal Stability of Wheeled Mobile Robots - Degree of Stability

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Abstract. **In this paper, the research focus is on the longitudinal stability of a wheeled mobile robot, using a geometric similarity coefficient (this coefficient is defined in the paper). The research method used for calculations is D'Alembert's principle. The results represent the limit driving/braking forces and limit accelerations/ decelerations for a given geometric similarity coefficient, before the robot loses stability.**

Keywords: Degree of stability, longitudinal stability, stability coefficient, wheeled mobile robots

I. INTRODUCTION

In this paper, problems related to the stability of wheeled mobile robots during movement are studied. They refer to the methods of determining the stability limits of a design scheme and the comparison of stability between two or more design schemes. For example, moving the center of mass forward improves longitudinal stability during acceleration, but it results in reduced stability during deceleration.

Usually, in similar studies, differential equations are used [1], [2], [4], [5] to represent the mathematical model. Instead, the D'Alembert's principle is used here, which considers an equilibrium system of forces, including inertial forces [6]; the accuracy of the calculations does not deteriorate. The wheels are assumed to contact the road at a point. Rolling friction forces are neglected. It is assumed that the motion occurs without slipping. It is also assumed that the robot body and its wheels are perfectly rigid, i.e. no deformations during movement. Lateral stability is not considered here.

The aim of the present study is to propose a method/approach for comparison of stability (of permissible forces and accelerations) of different design schemes of wheeled mobile robots.

To achieve the aim, we set ourselves the following tasks:

- to determine the parameters affecting the stability of the robot;
- the relationship between the robot's geometric parameters should be set quantitatively (with a number);
- to demonstrate through numerical experiments that the method provides an adequate estimate of the stability of the robot;
- to compare with examples the robot stability results by D'Alembert's method and by the proposed method.

Our hypothesis is that if the combination of values of the geometric parameters of one design scheme and the combination of values of the geometric parameters of another design scheme give the same values of their corresponding stability coefficients, then their permissible longitudinal forces (in case the mass of the constructions is equal) and accelerations, after which a loss of stability occurs of these two design schemes, are equal.

A criterion for loss of stability is the occurrence of a zero or negative value of the support reaction for any of the wheels of the robot.

The article is organized as follows.

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The second part contains an implementation and description of the mathematical model of the robot, as well as the notations used in the model. In the third part, specific values for the parameters of the robot, the constraints under which the experiments are carried out and the experimental results presented in a graphical form are given. In the last part, conclusions are made about the use of a stability coefficient and its relationship with limit forces and accelerations, after which a loss of stability occurs.

II. METHODS AND MATERIALS

A. Brief theory

Fig. 1 shows a longitudinal projection of the studied type of robot, which has rear wheel drive and four wheel brakes.

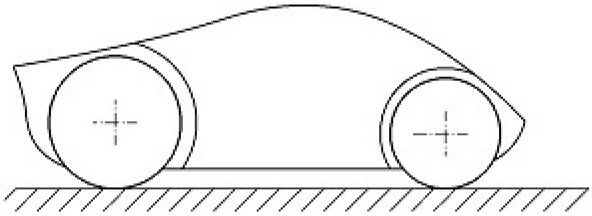

Fig. 1. Schematic of the robot in general view

The notations used in the mathematical model and those of Fig. 2, are shown below:

- O_{xz} – coordinate system related to the body of the robot;
- m_c – center of mass of the robot;
- A_r – position of the support points of the rear wheels;
- A_f – position of the support points of the front wheels;
- $l\ [m]$ – wheel base;
- $l_r\ [m]$ – distance along the x axis from the center of mass to the support points of the rear wheels;
- $l_f\ [m]$ – distance along the x axis from the center of mass to the support points of the front wheels;
- $z_{mc}\ [m]$ – distance from the mass center to the road;
- $m = 3\ [kg]$ – mass of the robot;
- $g = 9.807\ [m/s^2]$ – average ground acceleration (in the general case, the gravitational acceleration is a parameter);
- $a\ [m/s^2]$ – acceleration of the robot;
- $a_{dr}\ [m/s^2]$ – positive acceleration of the robot;
- $a_{br}\ [m/s^2]$ – deceleration of the robot;
- $a_p\ [m/s^2]$ – permissible acceleration of the robot;
- $a_{drp}\ [m/s^2]$ – permissible acceleration of the robot;
- $a_{brp}\ [m/s^2]$ – permissible deceleration of the robot;
- $F_g\ [N]$ – weight;
- $F_{dr}\ [N]$ – driving force;
- $F_{br}\ [N]$ – braking force;
- $F_{drp}\ [N]$ – permissible driving force;
- $F_{brp}\ [N]$ – permissible braking force;
- $F_{in}\ [N]$ – inertial force;
- $F_{indr}\ [N]$ – inertial force during acceleration;
- $F_{inbr}\ [N]$ – inertial force during deceleration;
- $F_{sr}\ [N]$ – support reaction at the rear wheels;
- $F_{sf}\ [N]$ – support reaction at the front wheels;

The mathematical model of the robot is built according to D'Alembert's principle, for which in this case it is necessary to compile a system of two moments and one projection equation according to the Fig. 2:

$$\left|\begin{array}{l} \sum M_{A_{r_i}} = 0 \\ \sum M_{A_{f_i}} = 0 \\ \sum x_i = 0 \end{array}\right. \tag{1}$$

$$\left|\begin{array}{c} F_g\, l_r + F_{in}\, z_{m_c} + F_{s_f}\, l = 0 \\ F_g\, l_f + F_{in}\, z_{m_c} + F_{s_r}\, l = 0 \\ F_{tr} + F_{in} = 0 \end{array}\right. \tag{2}$$

If the robot is equipped with the necessary sensors for reading the driving force and an actuator for supplying the required driving force F_{dr}, i.e. if F_{dr} is a parameter, only the support reactions of the wheels remain unknown in the system. The remaining terms in the equations are either constants or parameters:

$$F_g = m\, g \tag{3}$$

$$l = x_{A_f} - x_{A_r} \tag{4}$$

The following dependence determining $K_{base} = const$ is:

$$l_r = \frac{1}{3} l;\ l_f = l - l_r;\ z_{mc} = \frac{l}{6} \tag{5}$$

For the support reactions, during acceleration:

$$F_{s_f} = \frac{F_{dr}\, z_{m_c} - m\, g\, l_r}{l} \tag{6}$$

$$F_{s_r} = \frac{F_{dr}\, z_{m_c} + m\, g\, l_f}{l} \tag{7}$$

For the support reactions, at deceleration:

$$F_{s_f} = \frac{F_{br}\, z_{m_c} + m\, g\, l_r}{l} \tag{8}$$

$$F_{s_r} = \frac{m\, g\, l_f - F_{br}\, z_{m_c}}{l} \tag{9}$$

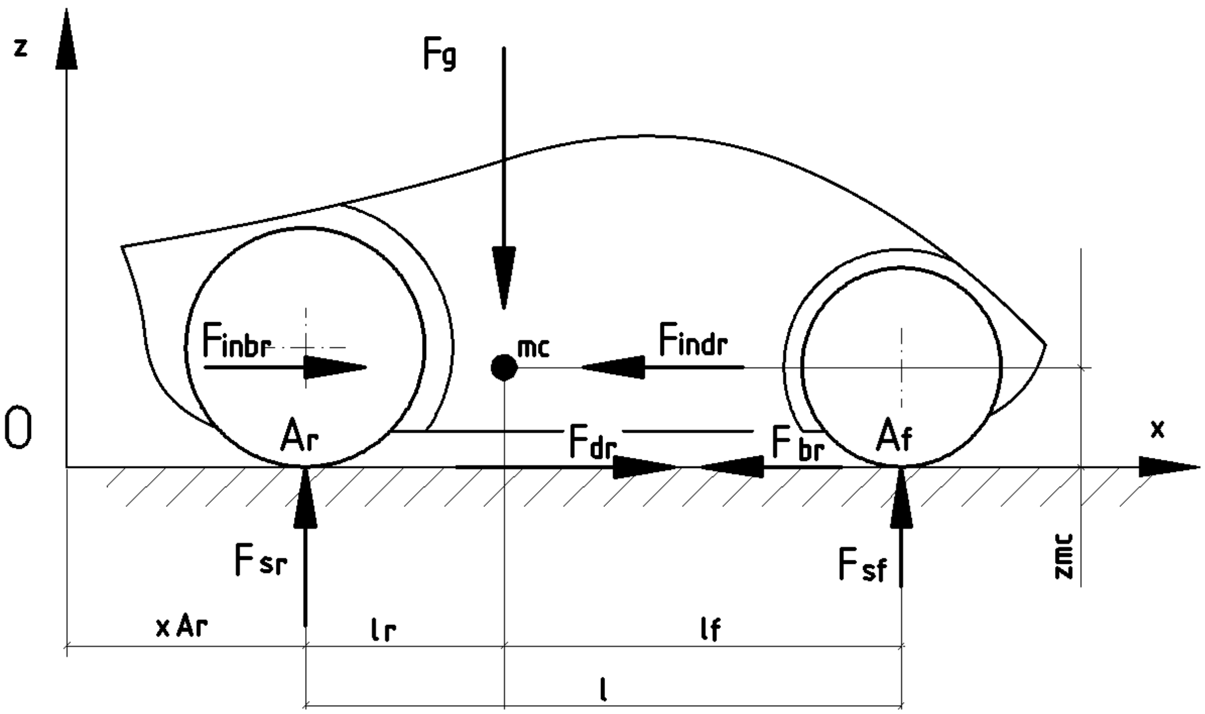

Fig. 2. Schematic of the robot with a coupled coordinate system and the acting forces (longitudinal projection)

According to the selected loss of stability criterion, in order to find the permissible driving and braking forces, under acceleration/deceleration, we assume the support reactions to be zero in the following two equations:

$$F_{dr_p} = \frac{m\,g\,l_r - F_{s_f}\,l}{z_{m_c}} \tag{10}$$

$$F_{dr_p} = \frac{m\,g\,l_f - F_{s_r} l}{z_{m_c}} \tag{11}$$

Because $F_{drp} = ma_{drp}$; $F_{brp} = ma_{brp}$, then again for support reactions equal to zero:

$$a_{drp} = \frac{m\,g\,l_r - F_{s_f}\,l}{z_{m_c}\,m} = \frac{g\,l_r}{z_{m_c}} - \frac{F_{s_f}\,l}{z_{m_c}\,m} \tag{12}$$

$$a_{brp} = \frac{m\,g\,l_f - F_{s_r}\,l}{z_{m_c}\,m} = \frac{g\,l_f}{z_{m_c}} - \frac{F_{s_r}\,l}{z_{m_c}\,m} \tag{13}$$

B. Implementation

The experiments are conducted for acceleration and deceleration.

The steps in performing the calculations are:

- The values for the constants are defined.

The ground acceleration g and mass m remain the same for all calculations.

- Defining $K_{base_r} = \frac{z_{mc}}{l_r}$;
- Defining $K_{base_f} = \frac{z_{mc}}{l_f}$;
- Defining $K_{base_1} = \frac{z_{mc}}{l}$;
- Defining $K_{base_2} = \frac{{z_{mc}}^2}{l_r l_f}$;
- Defining $K_{base_4} = \frac{{z_{mc}}^4}{{l_r}^2 {l_f}^2}$;
- K_{base} when $z_{mc} \in [0.05, 0.20]$; $l = 0.30$;
  - K_{base} when $z_{mc} \in [0.05, 0.30]$; $l = 0.60$;
- K_{base} when $z_{mc} \in [0.05, 0.60]$; $l = 1.20$;
- K_{base} when $l \in [0.15, 0.40]$; $z_{mc} = 0.10$ and l is along O_x axle;
- K_{base} when $l \in [0.15, 0.40]$; $z_{mc} = 0.10$ and l_r is along O_x axle;
- K_{base} when $l \in [0.15, 0.40]$; $z_{mc} = 0.10$ and l_f is along O_x axle;
- The force of gravity is determined according to the values of the earth's acceleration and mass: $F_g = mg$.
- For acceleration, the permissible driving force is calculated by the equation:

$$F_{dr_p} = mgl_r / z_{mc}.$$

- For deceleration, the permissible braking force is calculated using the equation:

$$F_{br_p} = mgl_f / z_{mc}.$$

- The permissible acceleration is:

$$a_{dr_p} = gl_r / z_{mc}.$$

- The permissible deceleration is:

$$a_{br_p} = gl_f / z_{mc}.$$

III. Experiments and results

A. Subject of experiments

Design scheme of a rear-wheel drive four-wheeled mobile robot with four wheel brakes is the subject of experiments. We assume that the robot is supplied with engine and brakes, which are capable to unbalance the robot during acceleration, respectively – deceleration.

B. Constrains

We accept the following restrictions:

- resistance forces as a result of contact with the road, friction in the robot units, air flow, etc. are not taken into account;
- the wheels contact the road at a point;
- the movement is non-slip;
- acting forces do not deform the robot units.

C. Results

The Octave 8.4.0 programming language was used to conduct the experiments.

The experiments are carried out on a design scheme of a four-wheeled rear-wheel drive mobile robot. Five variants of K_{base} are selected. K_{baser} gives a unambiguous comparison in acceleration. K_{basef} gives a unambiguous comparison on deceleration. K_{base1} is suitable when comparing design schemes which a certain route has to be passed. According to the characteristics of the terrain and according to the braking and acceleration capabilities, this coefficient can help to select an optimal location of the center of mass. But it does not give an unambiguous comparison specifically for acceleration and deceleration. K_{base2} and K_{base4} are similar to K_{base1}, but allow more strict control in design schemes with low stability due to their steeper graph, in the area of less stability. In other words, they can play the role of weighting coefficients in optimization tasks.

Normally, the coefficient K_{base} is chosen so that the closer its values are to zero, the greater the stability of the design scheme. In appropriate cases, such as when linear functions are required and we therefore want to free the denominator from variables, we can use the reciprocal, i.e. ${K_{base}}^{-1}$. Also, ${K_{base}}^{-1}$ is more intuitive because its value increases along with the value of F_{drp}; F_{brp}; a_{drp}; a_{brp}, but on the other hand, at K_{base} the maximum degree of stability is when $K_{base} \to 0$, i.e. to a certain value from the number axis.

The presence of some negative values of forces and accelerations in the results is due to their orientation relative to the coupled to the robot's body coordinate system.

Following are some explanations about the experiments and their relevant graphs presented.

a) Fig. 3, Fig. 4 and Fig. 5 illustrate the increase in K_{base} values when the stability decreases (when the height of the mass center z_{mc} increases, the stability decreases).

b) Conversely, in Fig. 6, Fig. 7 and Fig. 8, the values of K_{base} decrease as the stability increases (as the wheelbase l increases, respectively l_r and l_f, the stability increases).

c) In Fig. 9 and Fig. 10 again the dependence of K_{base} on the stability is given, but instead of indirectly, through the geometrical parameters of the dsign scheme, the permissible forces and accelerations are located directly along the Ox axis.K_{basef} is not applicable to F_{drp} and a_{drp}; K_{baser} is not applicable to F_{brp} and a_{brp}, so such graphs are not displayed.

d) Fig. 11 and Fig. 12 show the permissible forces and accelerations for design schemes with different stability, which increases with increasing l and decreases with increasing z_{mc}. This case can also be interpreted not as a comparison of different design schemes, but as a design scheme with a variable center of mass position and a variable wheelbase. Also here one can notice non-linear graphs when there is a variable in the denominator of the corresponding equations (10), (11), (12), (13).

e) In Fig. 13 it can be seen that the values of K_{base} depend on the relations between the geometric parameters of the design scheme, regardless of the absolute values of these parameters.

f) Fig. 14 shows that if we compare robots that have K_{base} with equal values, then their permissible accelerations will also be equal, and if the values of their masses m_i are equal too – and their permissible forces will also be equal (10), (11).

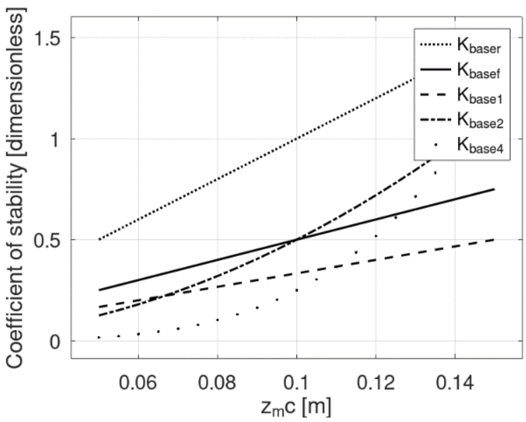

Fig. 3. Coefficients of stability according to z_{mc} value and $l = 0.3$, ($l_r = \frac{1}{3}l$; $l_f = l - l_r$).

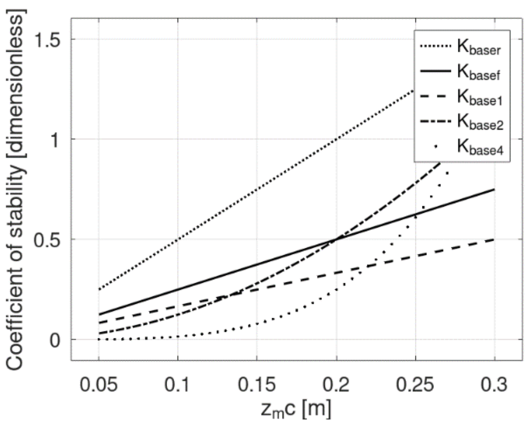

Fig. 4. Coefficients of stability according to z_{mc} value and $l = 0.6$, ($l_r = \frac{1}{3}l$; $l_f = l - l_r$).

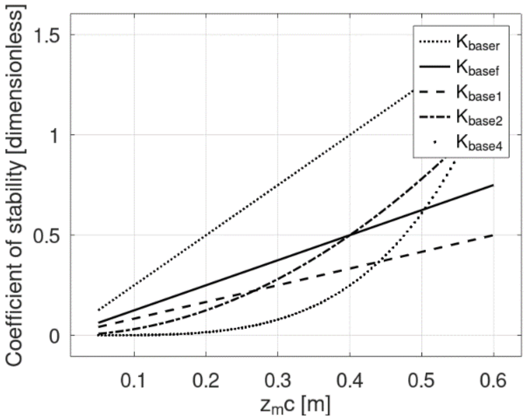

Fig. 5. Coefficients of stability according to z_{mc} value and $l = 1.2$, ($l_r = \frac{1}{3}l$; $l_f = l - l_r$).

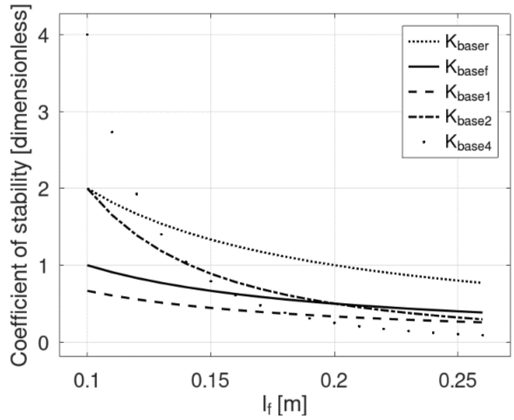

Fig. 8. Coefficients of stability according to l_f value and $z_{mc} = 0.1$, ($l_r = \frac{1}{3}l$; $l_f = l - l_r$).

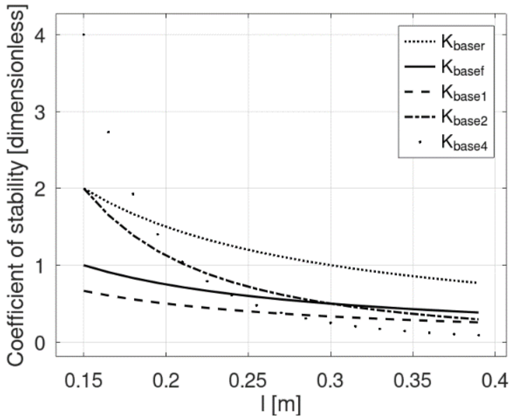

Fig. 6. Coefficients of stability according to l value and $z_{mc} = 0.1$, ($l_r = \frac{1}{3}l$; $l_f = l - l_r$).

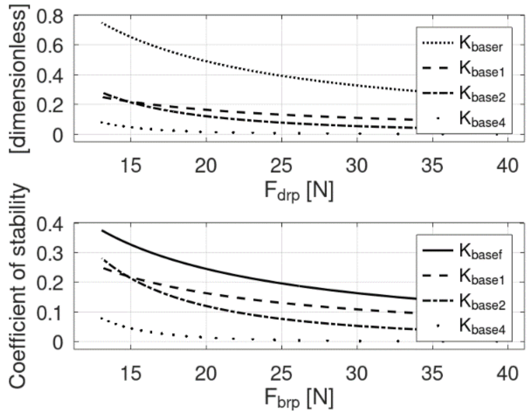

Fig. 9. Coefficients of stability according to F_{drp}; F_{brp} value and $z_{mc}\!: = 0.05\!:0.005\!:0.15$, ($l = 0.60; l_r = \frac{1}{3}l$; $l_f = l - l_r$).

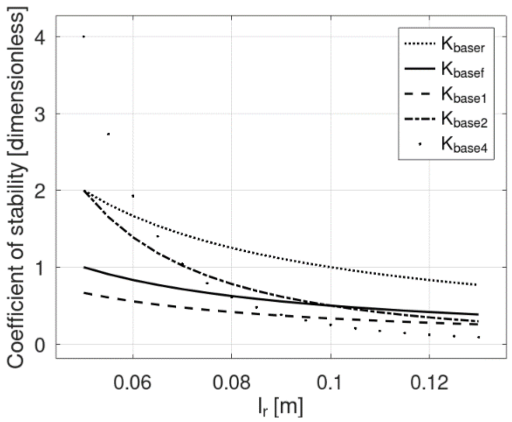

Fig. 7. Coefficients of stability according to l_r value and $z_{mc} = 0.1$, ($l_r = \frac{1}{3}l$; $l_f = l - l_r$).

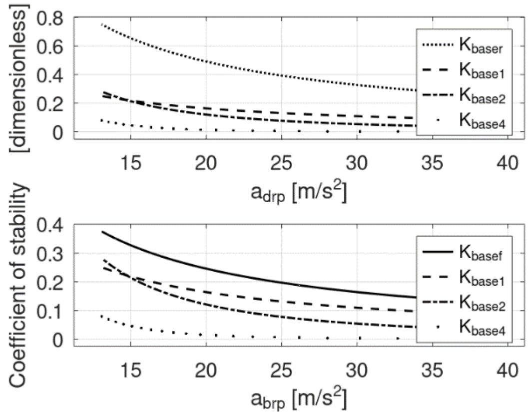

Fig. 10. Coefficients of stability according to a_{drp}; a_{brp} value and $z_{mc}\!: = 0.05\!:0.005\!:0.15$, ($l = 0.60; l_r = \frac{1}{3}l$; $l_f = l - l_r$).

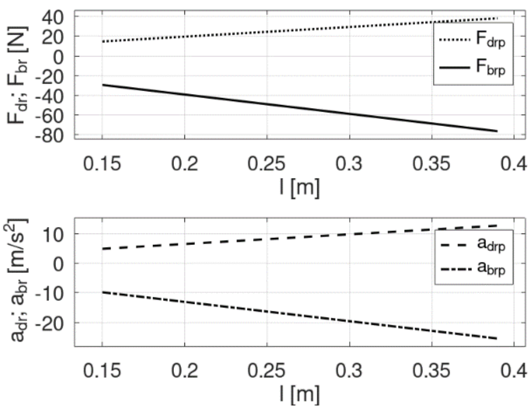

Fig. 11. Permissible forces and accelerations according to l value and $z_{mc} = 0.1$, $(l_r = \frac{1}{3}l;\ l_f = l - l_r)$, i.e. $K_{base} \neq const.$

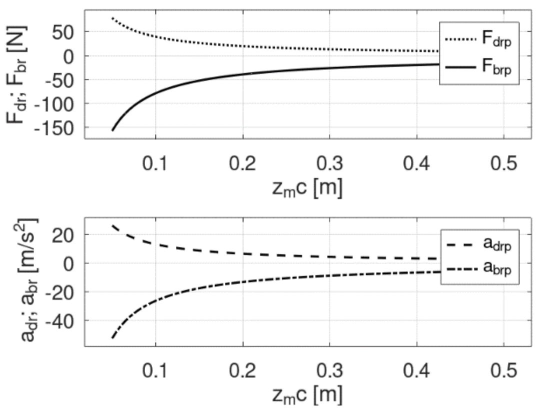

Fig. 12. Permissible forces and accelerations according to z_{mc} value and $l = 0.4$, $(l_r = \frac{1}{3}l;\ l_f = l - l_r)$, i.e. $K_{base} \neq const.$

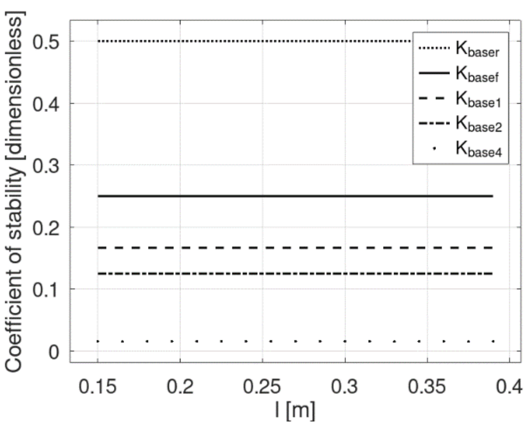

Fig. 13. K_{base} according to l value when $l_r = \frac{1}{3}l;\ l_f = l - l_r;\ z_{mc} = \frac{1}{6}l$, i.e. $K_{base} = const.$

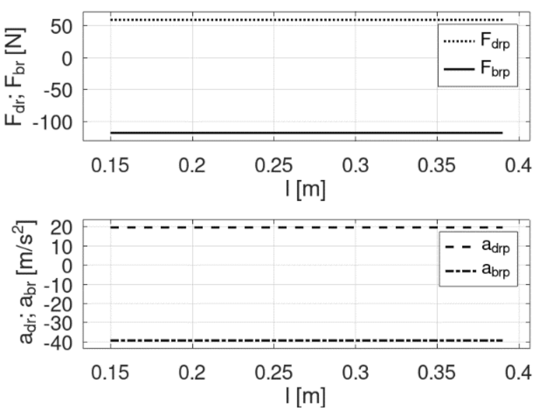

Fig. 14. Permissible forces and accelerations according to l value when $l_r = \frac{1}{3}l;\ l_f = l - l_r;\ z_{mc} = \frac{1}{6}l$, i.e. $K_{base} = const$ (see Fig. 13). But here $F = const$ only when $m = const$ "(10), (11)".

IV. CONCLUSION

The longitudinal stability of a design scheme of four-wheeled mobile robots with different values of the geometric similarity coefficient was studied. It was found that:

– the permissible driving/braking forces depend on the ratio between the individual geometrical parameters of the robots' design scheme, i.e. depend on the geometric similarity coefficient;

– the permissible driving/braking forces do not depend on the absolute values of the individual geometric parameters of the robots' construction, if the coefficient of geometric similarity is equal to a constant; in this case the permissible driving/braking forces will also be equal to a constant;

– the permissible accelerations/decelerations depend on the ratio between the individual geometrical parameters of the robots' design scheme, i.e. design schemes, having equal coefficients of geometric similarity have equal permissible accelerations/decelerations;

– the permissible accelerations/decelerations do not depend on the absolute values of the individual geometric parameters of the robots' design scheme, if the coefficient of geometric similarity is equal to a constant; in this case the permissible accellerations/decelerations will also be equal to a constant.

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