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Analytical model for determining the friction force at the contact of a metal body with a copper contact surface

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***Abstract*. The paper presents the development of an analytical model for calculating the frictional force when a copper contact surface slides across a static metallic body. Available analytical tribological models and mathematical methods were used to create the model. This model can be used to obtain information about the barrel wear of an artillery gun.**

Keywords: Analytical model, sliding motion, friction force, tribology.

I. Introduction

The motions of elements relative to each other is related to the occurrence of frictional forces. These frictional forces are an undesirable phenomenon in most machine elements, as they lead to material wear.

The tribology science deals with the study of frictional forces, the lubrication, the performance and reliability of machine elements. It is the basis for developing and implementing methods to increase the wear resistance of machine elements.

These methods are based on derived mathematical dependencies and analytical models to determine and research frictional forces. In the available literature on tribology, a wide variety of derived and proven dependencies and models for determining the values of friction forces at various contact interactions of machine elements are observed [2], [3], [6], [10], [11], [13], [16], [17], [18], [20], [22].

This dependence mainly affects general purpose machine elements. Models for determining the frictional forces of special-purpose machine elements are more difficult to reach. As a result, it is necessary to carry out a study of the contact characteristics of the researched special purpose machine elements and the available dependencies and models. In this way, it is possible to develop a useful analytical model for studying the research problem.

The artillery tube is one such a special purpose machine element. In the artillery tube, a frictional force occurs as a result of the motion of the projectile in their bore. The contact interaction is between the chrome-nickel coating of the inner surface of the artillery tube and the copper rotating band of the projectile [12].

The report presents an analytical model for evaluating and determine the frictional force that occurs in contact between artillery tube and projectile rotating band. The dependencies and models used are for the determination of friction forces and their calculation in the development of various machine elements and details.

II. Materials and Methods

In recent years, the dependences for determining the parameters of certain tribological variables have been increasing more and more. The wide variety of existing dependencies for calculating and determining frictional forces makes it almost impossible to cover them completely. Therefore, the report covers the main, most accessible and most frequently used dependencies for determining the tribological processes occurring at the contact of two elements, related to the studied problem.

Excluding the specific conditions of the studied problem and considering the motion of the projectile through the bore of artillery tube, as a contact scheme, it can be concluded that the following forces occurs in this process:

- sliding friction force;
- rotating friction force;
- frictional force from the flow of the gunpowder gases;

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- adhesion friction force.

For develop the model, analytical dependencies are used, which approach the contact scheme of the problem under research. These dependencies have been redone using general mathematical principles to more fully represent the researched frictional force.

In developing the model is used the same one characteristic, which corresponds to the same one variable. Accordingly, once written, the variable will not be written further in the paper.

Based on the analysis made of the available literature [2], [4], [5], [8], [9], [12], [19], [21], it is assumed that for the projectile-artillery tube contact scheme, hydrodynamic friction forces must be taken into account since there is presence of gunpowder gases moving between the two elements [2], [3], [6], [10], [13], [17], [18], [20]. Consideration of these hydrodynamic friction forces is imperative due to the fact that the presence of even a minimal amount of fluid significantly changes the nature of the friction forces.

Accordingly, in the determination of the dynamic force of sliding friction (occurring during the forward motion of the projectile), the widely accepted Newtonian relation for hydrodynamic friction can be used [18], [20]:

$$F_{sl} = \eta . S_a . G \quad [\mathrm{N}] \tag{1}$$

where: F_{sl} – sliding friction force [N];
η – dynamic viscosity of the fluid [Pa.s];
S_a – nominal contact area [m^2];
G – velocity gradient [s^{-1}].

Velocity gradient it can be determined by [18], [20]:

$$G = \frac{v}{h_d} \quad [\mathrm{s}^{-1}] \tag{2}$$

where: v – velocity of the linearly moving element [m/s];
h_d – clearance between the two elements in contact [m].

In addition, as a result of the viscosity of the fluid present in the contact between the two elements and their very rapid separation, the viscous friction force (friction force caused by the flow of the gunpowder gases) occurs. The dependency derived to calculate this force is [3]:

$$F_v = \frac{{h_d}^2 . \eta}{t_s} \quad [\mathrm{N}] \tag{3}$$

where: F_v – viscous friction force [N];
t_s – the required time to separate the two contact elements [s].

The analysis of the wear mechanism of the artillery tube shows that during the movement of the projectile in the bore of the gun tube, a frictional adhesion force also occurs. This friction force occurs as a result of the high-speed progressive and rotational motion of the projectile accompanied by high temperature and pressure of the burning gunpowder composition. In this process, the metal composition of the artillery tube coating and the rotating band of the projectile is liquefied, which can lead to adhering of surface asperity. In addition, the small clearance between the projectile and the artillery tube bore creates compression of their surface layers under the action of normal loading.

In the conditions of liquid-mediated contact, the determination of adhesion friction force can be done using the model of McFarlane and Tabor [3]:

$$F_a = \frac{\partial^2 . \eta}{t_s} \quad [\mathrm{N}] \tag{4}$$

where: F_a – adhesion friction force [N];
∂ – proportionality constant (dimension of length) [m^2].

The models described so far present information only about the individual frictional forces that occur in the contact of the projectile with the artillery tube.

In order to more accurately determine the friction forces occurring when two elements are in contact and to obtain more reliable data, it is necessary to collect the individual friction forces.

Thus, for example, if it is assumed that there is a negligible interaction between adhesion and deformation processes during sliding (dynamic friction force), they can be collected [18], [20] to obtain the total friction force, or:

$$F_p = F_a + F_d \quad [\mathrm{N}] \tag{5}$$

where: F_p – total friction force [N];
F_d – dynamic friction force [N].

In another part of the studied available literature sources, even with the slightest presence of any fluid, the following dependence is presented for determining the total friction force [3]:

$$F_p = \mu_p . (W + F_d) + F_v \quad [\mathrm{N}] \tag{6}$$

where: μ_p – coefficient of friction;
W – normal load [N].

As a disadvantage of this dependencies, it can be pointed out that they do not take into account all the frictional forces occurring in the contact scheme of the problem under research.

For the precise calculation and determination of the coefficient of friction, various dependencies have been derived that apply to specific contact elements under certain operating conditions. Thus, for example, with an adhesive friction force present at plastic deformation, the coefficient of friction can be determined by means of the equation [3]:

$$\mu_s = \frac{\tau_a}{H} \tag{7}$$

where: μ_s – coefficient of friction at condition of adhesion and plastic deformation;
τ_a – shear stress [N/m^2];
H – hardness of softer element [N/m^2].

To determine the shear stress, according to the Hertzian model, for a circular contact surface, the equation is used [3]:

$$\tau_a = 0.31.p_o \quad [\text{N/m}^2] \tag{8}$$

where: p_o – maximum contact pressure [N/m²].

The maximum contact pressure can be determined using the equation [3]:

$$p_0 = \left(\frac{6.W.E^{*2}}{\pi^3.\mathcal{R}^2}\right)^{\frac{1}{3}} \quad [\text{N/m}^2] \tag{9}$$

where: E^* – composite modulus of elasticity [N/m²];
$\pi = 3{,}14$ – Archimedes' Constant;
$\mathcal{R}$ – composite radius [m].

The normal load – W is determined using the equation [3]:

$$W = \frac{4.N.E^*.S_W^{\frac{3}{2}}}{3.\pi^{\frac{3}{2}}.\mathcal{R}} \quad [\text{N}] \tag{10}$$

where: N – number of asperities on the roughness;
S_w – real contact area [m²].

The following equation is used to determine the composite radius [3], [6]:

$$\frac{1}{\mathcal{R}} = \frac{1}{R_1} + \frac{1}{R_2} \quad [\text{m}] \tag{11}$$

where: R_1 – principal radii of curvature for the first element in the contact scheme [m];
R_2 – principal radii of curvature for the second element in the contact scheme [m].

To determine the composite modulus of elasticity, the following equation is given [3], [6], [18]:

$$E^* = \frac{1-\upsilon_1^2}{\text{E}_1} + \frac{1-\upsilon_2^2}{\text{E}_2} \quad [\text{N/m}^2] \tag{12}$$

where: E_1 – Young's modulus of elasticity for the first element in the contact scheme [N/m²];
E_2 – Young's modulus of elasticity for the second element in the contact scheme [N/m²];
υ_1 – Poisson's ratio for the first element in the contact scheme;
υ_2 – Poisson's ratio for the second element in the contact scheme.

When determining the real contact area, in case of plastic deformations, the following equation can be used [3], [18]:

$$S_w = \frac{W}{H} \quad [\text{m}^2] \tag{13}$$

Some of the dependencies reviewed so far have been analysed and presented in a source [23].

III. Results and discussion

In accordance with what has been reviewed so far and the analysis presented in [23], in the equation – 5 and 6 it is necessary to replace the dynamic frictional force with the frictional force caused by the sliding motion of the projectile in the gun tube bore. As a result, using equation 5 and 6 and collecting the individual frictional forces that occur in contact between the projectile and the inner surface of the artillery tube bore to determine the total frictional force is obtained:

$$F = \mu.(W + F_{sl}) + F_v + F_a \quad [\text{N}] \tag{14}$$

From the solution of the equation 11, to determine the compound radius is obtained:

$$\mathcal{R} = \frac{R_s.R_k}{R_s + R_k} \quad [\text{m}] \tag{15}$$

where: R_s – projectile radius [m].
R_k – artillery tube bore radius [m];

Substituting equations 10 and 15 into equation 13, to determine the real contact area is obtained:

$$S_w = \frac{\frac{4.N.E^*.S_W^{\frac{3}{2}}}{3.\pi^{\frac{3}{2}}.\mathcal{R}}}{H} \quad [\text{m}] \tag{16}$$

After substituting equations 12 and 15 into equation 16 and solving the resulting equation, to determine the real contact area is obtained:

$$S_w = \frac{9.\pi^3.\left(\frac{R_s.R_k}{R_s+R_k}\right)^2}{16.N^2.E^{*2}.H^2} \quad [\text{m}] \tag{17}$$

By substituting equations 12, 15 and 17 into equation 10 to determine, the normal load the following dependency is obtained:

$$W = \frac{4.N.\left(\frac{1-\upsilon_1^2}{\text{E}_1} + \frac{1-\upsilon_2^2}{\text{E}_2}\right)\frac{9.\pi^3.\left(\frac{R_s.R_k}{R_s+R_k}\right)^2}{16.N^2.E^{*2}.H^2}.}{3.\pi^{\frac{3}{2}}.\frac{R_s.R_k}{R_s+R_k}} \quad [\text{N}] \tag{18}$$

After performing a mathematical conversion, to determine the normal load is obtained:

$$W = \frac{3.\pi^{\frac{3}{2}}.R_k.R_s}{4.N.H^2.(R_s+R_k)} \quad [\text{N}] \tag{19}$$

To determine the sliding friction force, it is necessary to determine the nominal contact area. In accordance with this, the general principles of mathematics and the fact that the projectile has a complex shape, the nominal contact area should be divided into two parts - a conical part and a cylindrical part. Then, to determine the nominal contact area is obtained [7]:

$$S_a = (2.\pi.r_{cy}.l_{cy}) + (\pi.r_{co}.c) \quad [\text{m}^2] \tag{20}$$

where: r_{cy} – radius of the cylindrical part of the projectile [m];

l_{cy} – height of the projectile cylindrical part [m];

r_{co} – radius of the base of the projectile cone [m];

c – slant height of the projectile cone [m].

To determine the slant height of the projectile cone, the following equation is given [7]:

$$c = \sqrt{{r_{co}}^2 + l_{co}^2} \quad [\text{m}] \qquad (21)$$

where: l_{co} – height of the projectile cone [m].

Substituting equation 21 into equation 20, for determining the nominal contact area is obtained:

$$S_a = (2.\pi.r_{cy}.l_{cy}) + \left(\pi.r_{co}.\sqrt{{r_{co}}^2 + l_{co}^2}\right) \quad [\text{m}^2] \qquad (22)$$

It is necessary to select equation for determining the clearance between the projectile and the artillery tube bore. According to the deformation theory, it can be used [14]:

$$h = S_s - S_k = \pi.\left({R_s}^2 - {R_k}^2\right) \quad [\text{m}^2] \qquad (23)$$

where: $S_s = \pi {R_s}^2$ – projectile area [m^2];

$S_k = \pi {R_k}^2$ – the gun tube bore area [m^2];

By substituting equations 2, 22 and 23 into equation 1 to determine, the sliding friction force following dependency is obtained:

$$F_{sl} = \frac{\eta.\left[(2.\pi.r_{cy}.l_{cy}) + \left(\pi.r_{co}.\sqrt{{r_{co}}^2 + l_{co}^2}\right)\right].V_d}{\pi.\left({R_s}^2 - {R_k}^2\right)} \quad [\text{N}] \qquad (24)$$

where: V_d – projectile forward velocity [m/s];

To determine viscous friction force, it is seen from equation 3 that it is necessary to determine the time required for the projectile to pass through artillery tube. This can be obtained through [1], [15]:

$$t_1 = \frac{l_o}{V_d} \quad [\text{s}] \qquad (25)$$

where: t_1 – time required for the projectile to pass through artillery tube [s];

l_o – length of the rifled part of the artillery tube [m];

Substituting equations 23 and 25 into equation 3, to determine the viscous friction force is obtained:

$$F_v = \frac{\left[\pi.\left({R_s}^2 - {R_k}^2\right)\right]^2.\eta}{\frac{l_o}{V_d}} = \frac{\left[\pi.\left({R_s}^2 - {R_k}^2\right)\right]^2.\eta.V_d}{l_o} \quad [\text{N}] \qquad (26)$$

Substituting equation 25 into equation 4, to determine the adhesion friction force is obtained:

$$F_a = \frac{\partial^2.\eta.V_d}{l_o} \quad [\text{N}] \qquad (27)$$

Substituting equations 12, 15 and 19 into equation 9, and performing a mathematical conversion, to determine the maximum contact pressure is obtained:

$$p_0 = \left[\frac{(9.R_s + 9.R_k).\left(\text{E}_2 - \upsilon_1^2.\text{E}_2 + \text{E}_1 - \text{E}_1.\upsilon_2^2\right)^2}{2.\pi^{\frac{3}{2}}.\text{E}_1.H^2.R_k.N.R_s.{\text{E}_2}^2}\right]^{\frac{1}{3}} \quad [\text{N/m}^2] \qquad (28)$$

After substituting equation 28 into equation 8, to determine the shear stress is obtained:

$$\tau_a = 0.31.\left[\frac{(9.R_s + 9.R_k).\left(\text{E}_2 - \upsilon_1^2.\text{E}_2 + \text{E}_1 - \text{E}_1.\upsilon_2^2\right)^2}{2.\pi^{\frac{3}{2}}.\text{E}_1.H^2.R_k.N.R_s.{\text{E}_2}^2}\right]^{\frac{1}{3}} \quad [\text{N/m}^2] \qquad (29)$$

After substituting equation 29 into equation 7, for determining the coefficient of friction, is obtained:

$$\mu_s = \frac{0.31.\left[\frac{(9.R_s + 9.R_k).\left(\text{E}_2 - \upsilon_1^2.\text{E}_2 + \text{E}_1 - \text{E}_1.\upsilon_2^2\right)^2}{2.\text{E}_1.\pi^{\frac{3}{2}}.H^2.R_k.N.R_s.{\text{E}_2}^2}\right]^{\frac{1}{3}}}{H} \qquad (30)$$

Substituting equations 19, 24, 26, 27 and 30 into equation 14, the analytical model for determining the friction force at the contact of a metal body with a copper contact surface is derived:

$$F = \frac{0.31.\left[\frac{(9.R_s + 9.R_k).\left(\text{E}_2 - \upsilon_1^2.\text{E}_2 + \text{E}_1 - \text{E}_1.\upsilon_2^2\right)^2}{2.\pi^{\frac{3}{2}}.\text{E}_1.H^2.R_k.N.R_s.{\text{E}_2}^2}\right]^{\frac{1}{3}}}{H}.$$
$$.\left[\frac{3.\pi^{\frac{3}{2}}.R_k.R_s}{4.N.H^2.(R_s + R_k)} + \frac{\eta.\left[(2.\pi.r_{cy}.l_{cy}) + \left(\pi.r_{co}.\sqrt{{r_{co}}^2 + l_{co}^2}\right)\right].V_d}{\pi.\left({R_s}^2 - {R_k}^2\right)}\right] +$$
$$+ \frac{\left[\pi.\left({R_s}^2 - {R_k}^2\right)\right]^2.\eta.V_d}{l_o} + \frac{\partial^2.\eta.V_d}{l_o} \quad [\text{N}] \qquad (31)$$

IV. Conclusions

Through the derived model, it is possible to theoretically determine the friction force that occurs when a metal body contacts a copper contact surface, such as the inner surface of an artillery tube bore and the rotating band of a projectile.

The model derivation is a step in the process of developing an algorithm for determining artillery tube bore wear. Such an algorithm is necessary for theoretical research and studying related to the construction of artillery tube.

The development of an algorithm for determining the wear resistance of artillery tube requires a more in-depth analysis of the processes leading to the artillery tube wear. This algorithm should include sequentially coupled analytical models to determine and study the wear processes of the artillery tube.

Through this algorithm it can be possible to perform theoretical research. Research is an important stage of scientific work, as through them data can be collected from the obtained results.

As a result, by comparing the results with those obtained from empirical research and using statistical methods, the adequacy of the developed analytical model can be assessed.

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