ECOLOGICAL AND ECONOMIC PROBLEMS OF LINEAR PROGRAMMING

ЭКОЛОГО-ЭКОНОМИЧЕСКИЕ ЗАДАЧИ ЛИНЕЙНОГО ПРОГРАММИРОВАНИЯ

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Abstract. *The article presents the results of teaching the topic "Substantial models of linear programming" for the 3rd year students from the faculty of physics and mathematics of Pskov State University. Given topic contributes to a better understanding of the content of the problem of linear programming and demonstrates applied mathematics resources. The article analyzes the tasks, causing the greatest difficulty for students studying linear programming, and conclusions are made.*

Keywords: *dual task of linear programming, economic interpretation of duality, linear programming.*

Introduction

Human relationship with nature can be seen at all stages of its development. During the transition from traditional to industrial and, afterwards, to a postindustrial society, the problem of environmental pollution becomes more urgent. There are many types of pollutions that have different origins, and the impact on the world around us, all of them are distributed in the air, water and soil. Human activities acquire such proportions that basic principles of ecological balance are violated. Desertification, depletion of fresh water resource, sand ozone layer, the reduction in forest area, sand impoverishment of the gene pool – all of this is the result of human activities.

The principles of minimizing production waste constitutes a single concept, aimed at protection of the environment, improvement of the environmental situation in the region and sanitary conditions for the personnel maintaining the production. The mathematical model of such a problem is an optimization model, and, in particular, the task of linear programming.

Let’s consider one task about the environment.

<u>The task:</u> An optimal plan needs to be developed for cutting standard steel sheets, providing the output of the planned number of pieces of different types at the minimum total waste, if it is known that three kinds of different work pieces must be cut out of the batch of sheet steel. A steel sheet of standard sizes can be cut in three ways. A map of the layout corresponds to every possible way of cutting. Output in pieces of various types is known from the layout map, and waste area when cutting a steel sheet for each of the methods of cutting. The data is given in the table below.

Table 1.

These task

Types of pieces	Output of pieces of different types from layout maps (units)			Planned am out for the amount of units
	1	2	3	
1	1	4	0	240
2	1	0	4	200
3	1	1	0	120
Area of waste (m^2)	1,4	0,1	2,1	

What number of steel sheets needs to be cut in one way or another to minimize the waste?

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Solution:

Let's compose the mathematical model of the task. The limits in the task are set by the amount of different types according to the production plan. The efficiency function leads to finding the minimum waste while cutting.

$$\begin{cases} x_1 + 4x_2 \geq 240 \\ x_1 + 4x_3 \geq 200 \\ x_1 + x_2 \geq 120 \\ x_{1,2,3} \geq 0 \end{cases} \quad (1.1)$$

where x_i $(i = 1, 2. 3)$ - the amount of the initial material (steel sheets), which are necessary to be cut in any of three ways correspondingly.

$$f = 1{,}4x_1 + 0{,}1x_2 + 0{,}1x_3 \to \min \quad (1.2)$$

Let's compose a task dual to the given and solve it with a simplex method.

$$\begin{cases} y_1 + y_2 + y_3 \leq 1{,}4 \\ 4y_1 + y_3 \leq 0{,}1 \\ 4y_2 \leq 0{,}1 \\ y_{1,2,3} \geq 0 \end{cases} \quad (1.3)$$

where y_i $(i = 1,2,3)$ - dual variables.

$$g = 240\,y_1 + 200\,y_2 + 120\,y_3 \to \max \quad (1.4)$$

$$\begin{cases} y_1 + y_2 + y_3 + y_4 = 1{,}4 \\ 4y_1 + y_3 + y_5 = 0{,}1 \\ 4y_2 + y_6 = 0{,}1 \\ y_{1,2,3,4,5,6} \geq 0 \end{cases} \quad (1.5)$$

Table 2.

Solution of a task by a simplex method

Base	Absolute terms	y_1	y_2	y_3	y_4	y_5	y_6	Assessment
y_4	1,4	1	1	1	1	0	0	1,4/4=1,4
y_5	0,1	**4**	0	1	0	1	0	0,1/4=0,025
y_6	0,1	0	4	0	0	0	1	
g	0	240	200	120	0	0	0	
y_4	1,375	0	1	3/4	1	-1/4	0	1,375/1=1,375
y_1	0,025	1	0	1/4	0	1/4	0	
y_6	0,1	0	**4**	0	0	0	1	0,1/4=0,025
g	-6	0	200	60	0	-60	0	
y_4	1,35	0	0	3/4	1	-1/4	-1/4	1,35/0,75=1,8
y_1	0,025	1	0	**1/4**	0	1/4	0	0,025/0,25=0,1
y_2	0,025	0	1	0	0	0	1/4	
g	-11	0	0	60	0	-60	-50	
y_4	1,275	-3	0	0	1	-1	-1/4	
y_3	0,1	4	0	1	0	1	0	
y_2	0,025	0	1	0	0	0	1/4	
g	-17	-240	0	0	0	-120	-50	

As while solving a direct task, we are also revising the dual one, lets use a table and write an answer to the original task: $x^*(0;120;50)$. Let's calculate the value of the efficiency function with optimal plan x^*.

$$f = 1{,}4 \cdot 0 + 0{,}1 \cdot 120 + 0{,}1 \cdot 50 = 12 + 5 = 17.$$

Answer: the first way of cutting should not be used; using the second way of cutting you should cut 120 sheets and the third one – 50 sheets. Minimal waste – 17 units.

Materials and methods

An all-purpose method for solving linear programming problems is the simplex method. However, for a deeper understanding and identifying links between parameters of a problem, it is not enough to solve a problem only with the simplex method. You often need to understand how changes of individual parameters of the problem may result in the solution, etc. That is why the concept of "economic interpretation of duality" plays an important role in the identification of such relationships.

I have developed abstracts and held classes on the topics of "dual linear programming problems" and "economic interpretation of duality." [1] In the classroom, the third year students of physics and mathematics (profile Applied Informatics) got acquainted with basic definitions, the rule of formation of a dual pair, duality theorems and properties of dual assessments. All theoretical aspects were given with the examples and solutions for various types of tasks.

In the final lesson, the students were asked to write the test.

Test consisted of two tasks. The students had to create a task, dual to the given task in the first task.

Example of task 1:

$$\begin{cases} x_1 + 2x_3 - 12x_4 + 7x_5 \geq 13 \\ x_2 + 7x_3 + 5x_4 + 6x_5 = 7 \\ x_1 + 4x_4 - 7x_5 \leq 9 \\ x_{1,2,4} \geq 0 \\ x_{3,5} - \forall \end{cases} \quad (2.1)$$

$$f = x_1 + 2x_2 + 8x_3 - 15x_4 + x_5 \to \min \quad (2.2)$$

Answer:

$$\begin{cases} y_1 - y_3 \leq 1 \\ y_2 \leq 2 \\ 2y_1 + 7y_2 = 8 \\ -12y_1 + 5y_2 - 4y_3 \leq -15 \\ 7y_1 + 6y_2 + 7y_3 = 1 \\ y_{1,3} \geq 0, y_2 - \forall \end{cases} \quad (2.3)$$

$$g = 13y_1 + 7y_2 - 9y_3 \to \max \quad (2.4)$$

In the second task they were offered to make a mathematical model, and find a solution to the dual task with a simplex method, solve the given problem of linear programming and economically interpret it.

Example of task 2:

Table 3.

These task

Resource	The rate of raw materials cost per unit of output			The amount of resources
	A	B	C	
1	2	1	1	26
2	7	1	2	84
3	1	8	1	78
Price for the unit of production	15	11	16	

Results and discussions

The first task was copmpleted by 75% of students, 25% of works were not credited. There were the following mistakes: while composing the dual task student didn't take into account, that there should be a specific relationship between the sign of the inequalities in constrains and the striving of the efficiency function to maximum or minimum. Besides, 86% of students forgot the fact the fact that each inequality in the set of constrains in a direct task has a corresponding variable y_i, without constrains to the sign in the dual task, and vice versa, and constraints for the inequalities in the dual task correspond to the negative variables x_k in the direct task.[2]

While solving the task students experienced some difficulties while composing economic-mathematical model of the task and analyzing sensitivity of the task.

Besides the test students were offered to participate in the anonymous survey.

Based on the survey we can make the following conclusions: 76% of students had difficulties while solving the problems of linear programming. As a rule, difficulties were connected with the understanding the text of the task, designing the mathematical model (translation of the task to the mathematical language) and with the task itself.

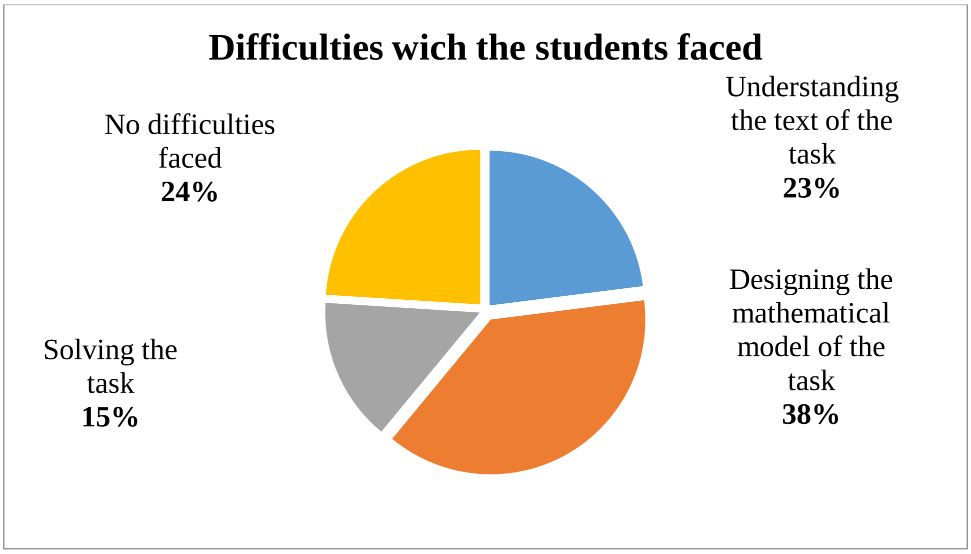

Fig. 1. **Chart of results**

There was a question in the survey as well: do these types of tasks help to apply obtained mathematical knowledge in the real life?

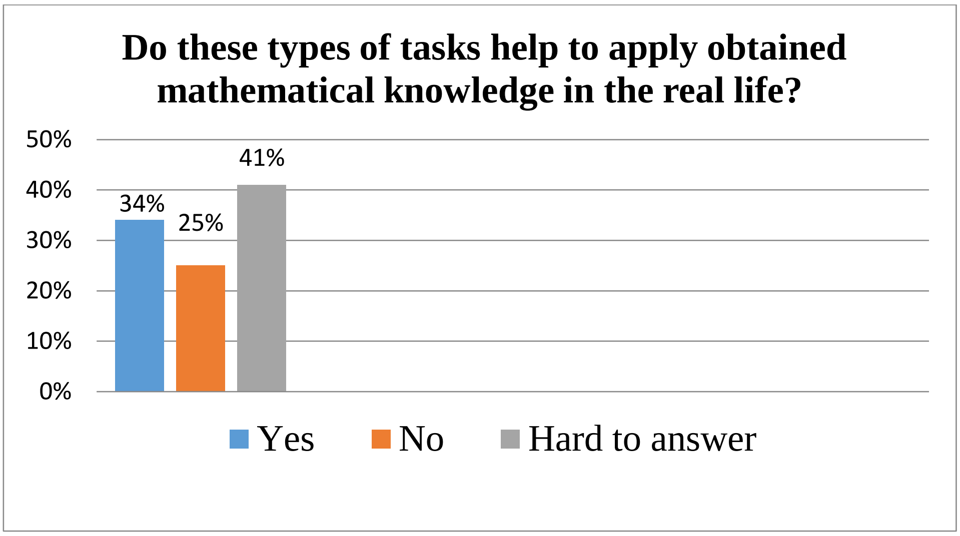

Fig. 2. **Chart of results**

Conclusions

I believe, that studies designed by me helped students to understand deeper linear programming problems, showed the applied mathematical resources while solving the tasks, explained the relationship between two dual tasks and helped in better understanding of interpretation of duality.

References

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