OPTIMAL ROUTE DETECTION BETWEEN EDUCATIONAL INSTITUTIONS OF REZEKNE MUNICIPALITY

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Abstract. *Information about the merger of schools or their optimization periodically appears in the society. It is believed that an ideal school network is not yet ready. The paper provides an analysis of the locations of educational institutions by their availability. Theoretical research has been conducted and mathematically the shortest route has been calculated among different educational institutions. The paper also provides mapping of these educational institutions and a location analysis of the educational institutions at different levels. The main goal of the paper is to show the possibilities of applying mathematical models in solving practical tasks – to determine the shortest route among the educational institutions. This study describes an optimization method called Simulated Annealing. The Simulated Annealing method is widely used in various combinatorial optimization tasks. Simulated Annealing is a stochastic optimization method that can be used to minimize the specified cost function given a combinatorial system with multiple degrees of freedom. In this paper, the application of the Travelling Salesman Problem is demonstrated, and an experiment aimed to find the shortest route among the educational institutions of Rezekne Municipality is performed. It gives possibilities to analyse and search an optimal school network in Rezekne Municipality. Common research methods are used in this research: the descriptive research method, the statistical method, mathematical modelling.*

Keywords: *Educational institutions, Rezekne Municipality, Optimization, Travelling Salesman Problem Simulated Annealing.*
JEL code: *C63*

Introduction

Information about school optimization periodically appears in the Latvian society. It is believed that the ideal school network is not ready. The Ministry of Education and Science commissioned the company "Jāņa Sēta" to perform the mapping of educational institutions. In a follow-up study, in Bauska Municipality, there can be seen both students' routes and the most populous and economically most active municipalities, that helped the municipality authority take the decision that secondary schools should be retained only in Bauska. When "Jāņa Sēta" develops a similar map for all of Latvia, then the ideal network of schools will be seen (Kuzmina, 2016). The paper only looks at basic schools and secondary level schools. The children

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of Rezekne Municipality have possibility to choose among 13 basic schools and 6 secondary schools. After school they can attend: one sports school for children and youth, one art school for children or one centre for children and youth ("Educational institutions," 2016). The paper offers the analysis of the locations of educational institutions based on their availability. The theoretical study has been carried out and the shortest route among different educational institutions is calculated mathematically, the mapping of these educational institutions and different levels of analysis of the locations of educational institutions are performed (e.g. to find the opportunities for basic school or kindergarten graduates to have access to education as close as possible to the place of residence) (the authors used data from ("Educational institutions," 2016).

The authors define the following levels of educational institutions:

Level 1: Primary schools; Level 2: Secondary schools.

The authors have developed the software that allows finding the shortest route among various educational institutions in Rezekne Municipality with the purpose to optimize and determine the shortest route among the educational institutions. A multi-tiered architecture and overlapping in the characterization of educational institutions is offered.

The main aim of the research study could be the development of recommendations and an analysis of the potential educational network optimization. The research study was carried out using Visual Studio capabilities in programming.

Common research methods are used in this research: the descriptive research method, the statistical method, mathematical modelling.

Theoretical background of the research study

Simulated Annealing (SA) is a stochastic optimization method used for the optimization of an objective function (energy). It allows finding the global extreme for the function that has local minimums. The SA principle was announced in a classical work (Kirkpatrick et al., 1983) and developed in other works (Laarhoven & Aarts, 1987), (Otten & Ginneken, 1987), (Granville et al., 1994), (Ingber, 1993).

SA is based on the analogy of statistical mechanics and, in particular, the solid-state physics elements. A practical example from metallurgy can be given – what happens to the atomic structure of the body if lowering its temperature or, in other words, if it is rapidly cooled. Rapid temperature reduction can lead to an unsymmetrical system structure, or in other words, to a sub-optimal position (with errors). Cooling ultimately leads to a condition where the system curdles or freezes, and thermal equilibrium sets in.

The so-called Metropolis procedure (Kirkpatrick et al., 1983) determines iterative steps, which control the best solution to be achieved. This algorithm is used in atomic equilibrium simulation at the given temperature. On each step of the algorithm, the atom is raised by a small probabilistic movement (shifting): $x_i + \zeta$, and system energy change ΔE is calculated.

- If $\Delta E \leq 0$, then the movement is accepted and configuration with altered states of atoms is used as the initial state for the next step.
- If $\Delta E > 0$, then the probability that the new state will be accepted is:

$$P(\Delta E) = e^{-\frac{\Delta E}{kT}} \tag{1}$$

where k – Boltzmann's constant, T– temperature parameter.

Using the energy system as a target function and defining the states of the system with $\{x_i\}$, it is seen that the Metropolis procedure generates a series of states for the given optimization problem at a particular temperature.

To use the SA method practically, the following must be specified:

1. The target function W (analogous to energy surface), whose minimization is the purpose of this procedure.
2. A possible set of solutions according to the energy surface or the physical state of the system.
3. Configuration conditions, the variation generator.
4. The control parameter T, which characterizes an artificial system temperature, and the cooling mode (annealing schedule) that describes how the temperature will be lowered.

The SA algorithm is based on the Boltzmann's probability distribution:

$$\Pr(e) \sim e^{-\frac{E}{kT}} \tag{2}$$

This expression specifies that if the system is in thermal equilibrium at a temperature T, then its energy is probably divided among all the different energy states E. Even at low temperatures, there is a possibility that the system may be found in a high energy state. The system has an adequate probability of moving from a local energy minimum state to a better, more global, minimum.

Further, as the SA algorithm application, the well-known combinatorial task will be offered – the Traveling Salesman Problem (TSP).

The TSP task is to find the minimum route among N cities – entering into each city only once and, in the end, returning to the original city. This is a well-known combinatorial task that can be solved with a variety of combinatorics or graph theory techniques. In literature TSP solving methods with the SA algorithm are also provided (Cook, 2011), (Coughlin & Baran, 1985), (Applegate et al., 2006), (Grabusts, 2000).

Let us define the distance matrix $D = (d_{ij}), i, j = 1,2, \dots, n,$ - distance between cities i and j. Each route can be represented as an element π of all permutations among n city sets. If a possible route set consists of all the cyclical permutations, then in total there are $\sqrt{(n-1)!}$ such permutations. The objective function is defined as follows:

$$C(\pi) = \sum_{i=1}^{n} d_{i\pi(i)} \tag{3}$$

The TSP task is to minimize the objective function in all possible permutations. If n cities are located in a 2-dimensional Euclidean space and d_{ij} is a Euclidean distance between cities i and j, then $C_{opt}^{(D)}$ is the shortest route for a given distance matrix D.

To use the SA algorithm for such a type of tasks, some concepts have to be introduced. For each route we can define the neighbour as a route set that can be reached from the current route during one transition. Such a neighbouring structure mechanism for the TSP is called k-opt transitions. In the simplest case – a 2-opt transition is based on the fact that two cities are selected on the current route and the sequence in which the cities between these pairs were visited, is reversed (see Figure 1).

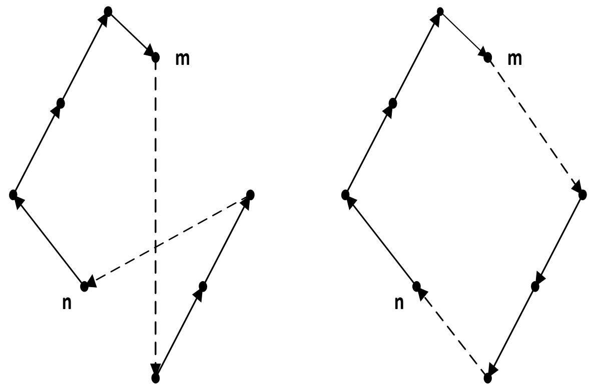

Fig.1 2-opt example (on the left – the current route, on the right – after reversing the sequence between m and n
(Source: authors' construction)

Route neighbours are now defined as a set of cities that can be reached from the current route through the 2-opt transitions (i.e. $\sqrt{(n-1)n}$) such neighbours).

Experimental research

In the research, different levels of educational institutions, educational institutions and their GPS coordinates were defined, the shortest route among educational institutions was computed by means of the SA algorithm and the depiction of educational institutions on geographic maps was carried out.

Level 1. Primary schools (see Table 1.)

Table 1 Denotation and GPS coordinates of primary schools *(Source: Google Maps)*

No.	Name of primary schools (in Latvian)	Latitude	Longitude
1	Audriņi *(Audriņu pamatskola)*	56.587559	27.242635
2	Bērzgale *(Bērzgales pamatskola)*	56.629493	27.516288
3	Feimaņi *(Feimaņu pamatskola)*	56.272112	27.042613
4	Gaigalava *(Gaigalavas pamatskola)*	56.734361	27.06622
5	Jaunstrūžāni *(Jaunstrūžānu pamatskola)*	56.695701	27.235483
6	Kruķi *(Kruķu pamatskola)*	56.405302	27.00685
7	Liepas *(Liepu pamatskola)*	56.419436	27.206095
8	Rēzna *(Rēznas pamatskola)*	56.435283	27.552322
9	Rikava *(Rikavas pamatskola)*	56.622145	27.044503
10	Sakstagals *(Sakstagala Jāņa Klīdzēja pamatskola)*	56.534155	27.144494
11	Verēmi *(Verēmu pamatskola)*	56.574573	27.366389

The SA algorithm, in this case, was carried out in 22 steps. The shortest route computed by means of the algorithm was 251 km (see Figure 2). The depiction of educational institutions on the map is shown in Figure 3.

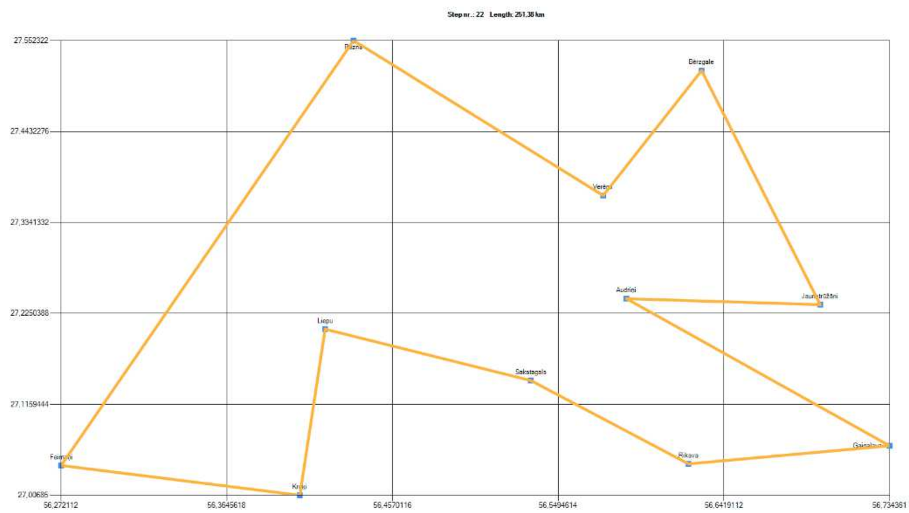

Fig.2 The shortest route among primary schools computed by means of the SA algorithm *(Source: authors' construction)*

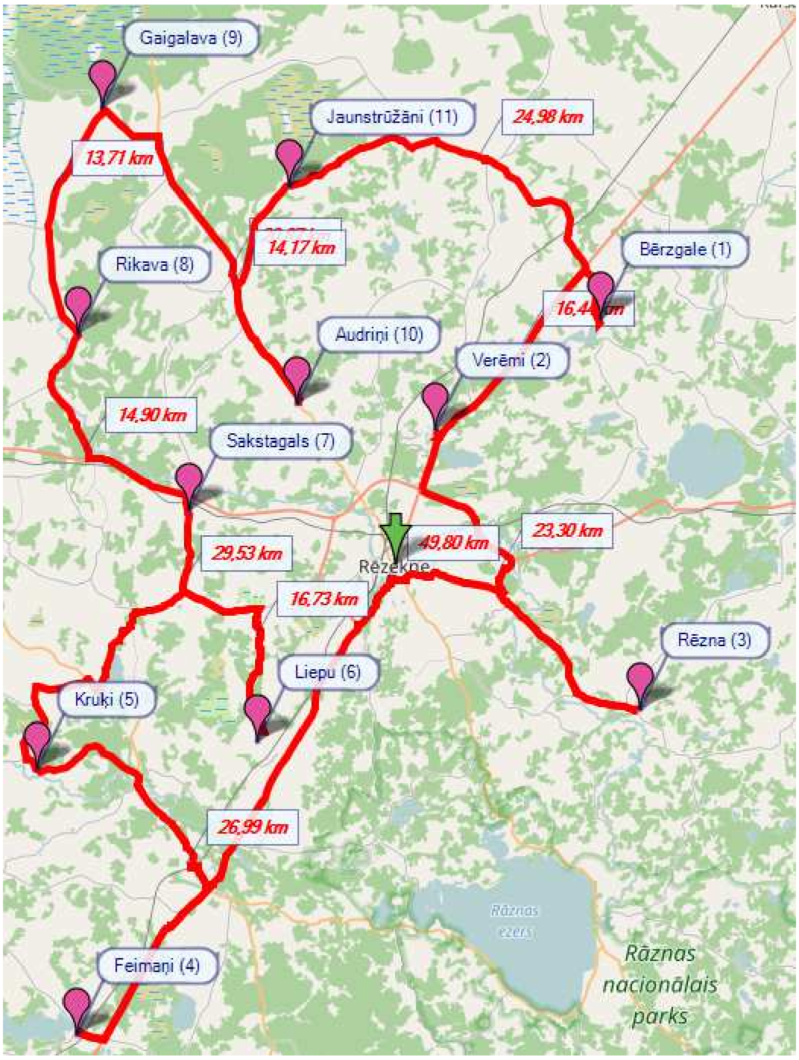

Fig.3 Depiction of the shortest route among primary schools on Google Maps *(Source: authors' construction)*

Level 2. Secondary schools (see Table 2)

Table 2 Denotation and GPS coordinates of secondary schools *(Source: Google Maps)*

No.	Name of secondary schools (in Latvian)	Latitude	Longitude
1	Dricāni (*Dricānu vidusskola)*	56.649232	27.182524
2	Kaunata (*Kaunatas vidusskola)*	56.331737	27.543208
3	Makašāni (*Lūcijas Rancānes Makašānu Amatu vidusskola)*	56.587671	27.315964
4	Malta (*Maltas vidusskola)*	56.347054	27.157439
5	Nautrēni (*Nautrēnu vidusskola)*	56.71153	27.412196
6	Tiskādi (*Tiskādu vidusskola)*	56.405377	27.007207

The SA algorithm, in this case, was carried out in 17 steps. The shortest route computed by means of the algorithm was 162 km (see Figure 4). The depiction of educational institutions on the map is shown in Figure 5.

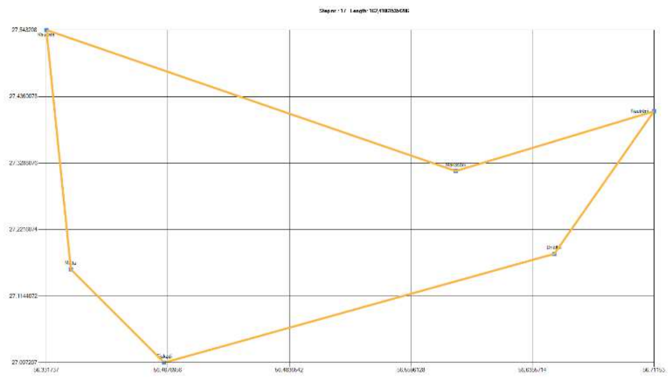

Fig. 4 The shortest route among secondary schools computed by means of the SA algorithm *(Source: authors' construction)*

Fig.5 Depiction of the shortest route among secondary schools on Google Maps *(Source: authors' construction)*

Similarly, statistics for 19 kindergartens and 3 special boarding primary schools was collected. How can it be used practically? Supposing that there is a need to find the optimal distance among secondary schools and primary schools, these educational institutions have to be depicted on the map with the purpose to analyze the potential children closeness to the educational institution. (see Figures 6 and 7).

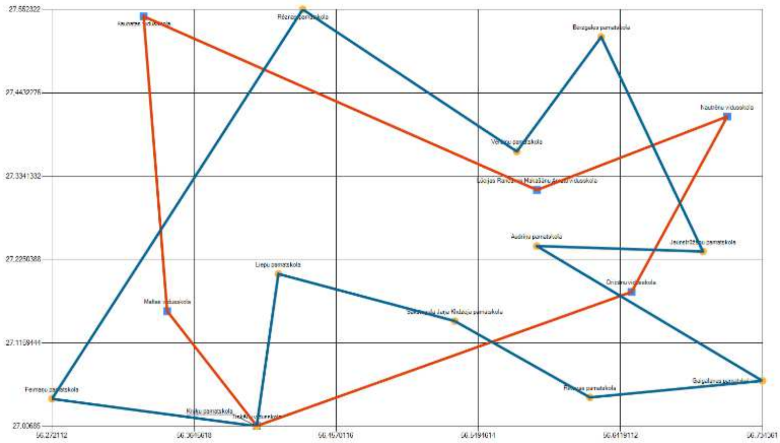

Fig. 6 The shortest route among primary schools and secondary schools computed by means of the SA algorithm *(Source: authors' construction)*

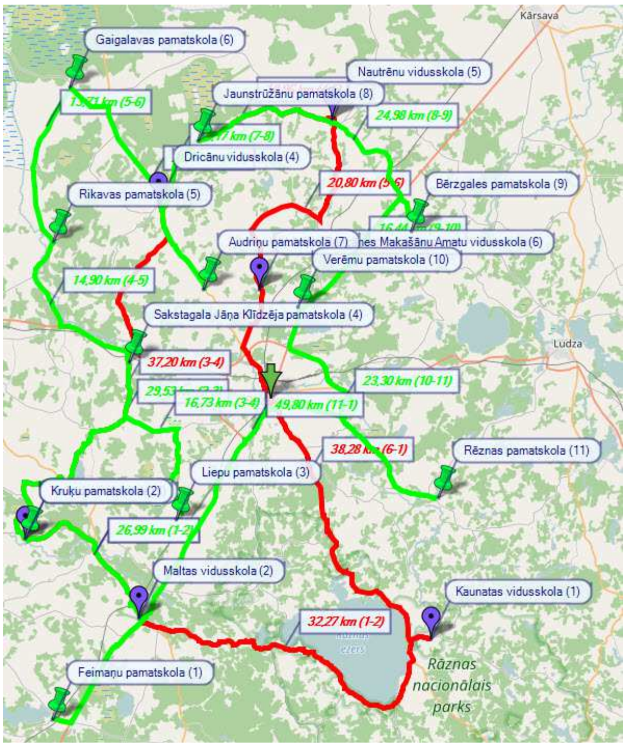

Fig. 7 Depiction of the shortest route amoung the two groups of educational institutions on Google Maps *(Source: authors' construction)*

The authors assume that the result is relatively simple for the heads of educational institutions. It is clear that for children, after finishing Jaunstrūžāni or Audriņi primary school, it is nearer to get education in Dricāni secondary school. But in this case, a theoretical modeling tool is offered, where one of the educational institutions is hypothetically excluded from the education system.

Conclusions

The authors proposed that their simulation result should be simplified, but in case it is needed to exclude a school from the existing network of educational institutions, it would allow simulating the overlapping of educational institutions on the map and determining the potentially shortest route to the chosen educational institution for children.

In this paper the software that allows finding the shortest way or route among different educational institutions in Rezekne Municipality with the purpose to optimize and determine the shortest route among educational institutions has been developed. The aim of the research study was to develop a modelling tool for analysis of potential educational network optimization. Unfortunately, the results of the research paper raise an interest neither for the Rezekne District Council nor for the publishing house "Jāņa sēta".

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