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SHORTEST PATH DETERMINATION BETWEEN EDUCATIONAL INSTITUTIONS OF RĒZEKNE MUNICIPALITY

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Abstract. *This study describes an optimization method called Simulated Annealing. The Simulated Annealing method is widely used in various combinatorial optimization tasks. Simulated Annealing is a stochastic optimization method that can be used to minimize the specified cost function given a combinatorial system with multiple degrees of freedom. In this study the application of the Simulated Annealing method to a well - known task of combinatorial analysis, Travelling Salesman Problem, is demonstrated and an experiment aimed to find the shortest tour distances between educational institutions of Rēzekne Municipality is performed. It gives possibilities to analyze and search optimal schools' network in Rēzekne Municipality.*
Keywords: *Educational institutions, Optimization, Rēzekne Municipality, Simulated Annealing, Travelling Salesman Problem.*

Introduction

Information about school consolidation or optimization periodically appears in Latvian society. It is believed that the ideal school network is not ready. Ministry of Education and Science offered the company "Jāņa Sēta" to develop the mapping of educational institutions. In a follow-up study, in Bauska county, there can be seen both students' pathways and the most populous and economically most active counties, that helped the county authority take the decision that secondary schools should be retained only in Bauska. When "Jāņa Sēta" develops a similar mapping for all of Latvia, then the ideal network of schools will be seen (Kuzmina, 2016). Children of Rēzekne Municipality have possibility to choose among 13 basic schools, 6 secondary schools, 3 special boarding basic schools and 19 kindergartens. After school they can attend: one sports school for children and youth, one art school for children or one center for children and youth ("Educational institutions," 2016). The article offers the analysis of the location of educational institutions based on their availability. The theoretical study has been carried out and the shortest path between different

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educational institutions is calculated mathematically, the mapping of these educational institutions and different levels of analysis of the location of educational institutions are offered (e.g. to find up the opportunities for basic school or kindergarten graduates have access to education as close as possible to the place of residence) (we used data from ("Educational institutions," 2016).

We define the following levels of educational institutions:
Level 1: Kindergartens; Level 2: Basic schools; Level 3: Secondary schools; Level 4: Special boarding basic schools; Level 5: Vocational oriented educational institutions.

We have developed the software that allows to find the shortest path between different educational institutions in Rēzekne Municipality with a purpose to optimize and determine the shortest route between educational institutions. A multi-tiered architecture in educational institutions characterization and overlapping is offered. The aim of the study could be the development of recommendations and analysis in the potential educational network optimization.

Mathematical background

Simulated Annealing (SA) is a stochastic optimization method used for the optimization of objective function (energy). It allows to find the global extreme for the function that has local minimums. SA principle was announced in the classical work (Kirkpatrick et al., 1983) and developed in works (Laarhoven & Aarts, 1987), (Otten & Ginneken, 1987), (Granville et al., 1994), (Ingber, 1993).

SA is based on the analogy of statistical mechanics and, in particular, the solid-state physics elements. The practical example from metallurgy can be given- what happens to the atomic structure of the body, lowering its temperature, in other words, if it is rapidly cooled. Rapid temperature reduction can lead to unsymmetrical system structure, or in other words, to a sub-optimal position (with errors). Cooling ultimately leads to a condition where the system curdles or freezes, and thermal equilibrium sets in.

The so-called Metropolis procedure (Kirkpatrick et al., 1983) determines iterative steps, which control the best solution to be achieved. This algorithm is used in atomic equilibrium simulation with the given temperature. On each step of the algorithm atom is raised with a small probabilistic movement (shifting): $x_i + \zeta$, and system energy change ΔE is calculated.

- If $\Delta E \leq 0$, then the movement is accepted and configuration with altered states of atoms is used as the initial state for the next step;

- If $\Delta E > 0$, then the probability that the new state will be accepted is:

$$P(\Delta E) = e^{-\frac{\Delta E}{kT}} \tag{1}$$

where k – Boltzmann's constant, T – temperature parameter.

Using the energy system as a target function and defining the states of the system with $\{x_i\}$- it is seen, that the Metropolis procedure generates a series of states for the given optimization problem with particular temperature.

Another way to understand SA as a combinatorial optimization method is to imagine the energy surface, as it is shown in Figure 1.

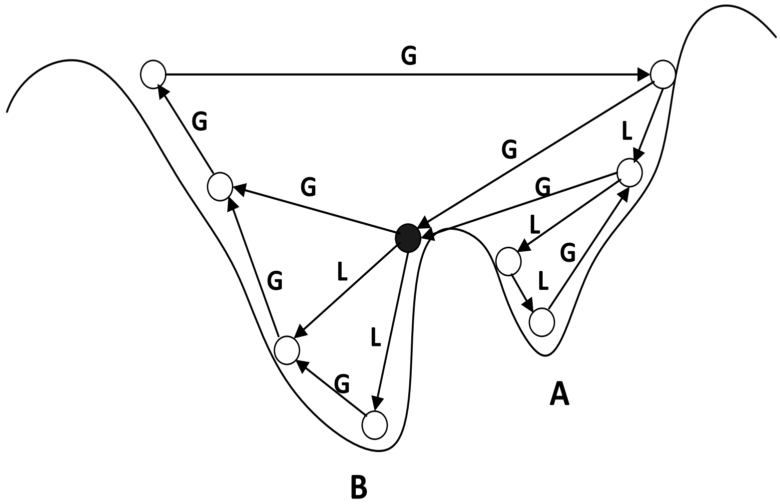

Fig.1 **Energy surface (G – global, L – local)**

The black globule, starting from the arbitrarily selected point, always searches for the way down. If such a system is compromised - and somehow is exposed (e.g. by shaking), then the globule will mostly move from A to B, because the energy barrier is less from the A side.

If there is a slight effect, then, obviously, the globule more often will move from A to B, not from B to A. If the effect is strong, then the globule will overcome the barriers faster and more frequently, that is it may move from A to B and from B to A. If, however, we want to affect the globule movement, then a good compromise would be to start with a stronger effect and gradually reduce the exposure. This will ensure that at some step the globule will pass the global minimum.

To use the SA method practically, the following must be specified:

1. Target function W (analogous to energy surface), whose minimization is the purpose of this procedure;
2. Possible set of solutions according to the energy surface or the physical state of the system;
3. Configuration conditions, the variation generator;
4. Control parameter T, which characterizes an artificial system temperature, and the cooling mode (annealing schedule), that describes how the temperature will be lowered.

SA algorithm is based on the Boltzmann's probability distribution:

$$\Pr(e) \sim e^{-\frac{E}{kT}} \tag{2}$$

This expression specifies that if the system is in thermal equilibrium with temperature T, then its energy is probably divided among all the different energy states E. Even at low temperatures there is a possibility that the system may be found in a high energy state. The system has an adequate probability of moving from a local energy minimum state to a better, more global, minimum.

Further, as SA algorithm application the well-known combinatorial task will be offered- the Traveling Salesman Problem (TSP).

Classical Travelling Salesman Problem

TSP task is to find the minimum route between N cities – entering into each city only once and in the end returning to original city. This is well-known combinatorial task that can be solved with a variety of combinatorics or graph theory techniques. In literature TSP solving methods with the SA algorithm are viewed also (Cook, 2011), (Coughlin & Baran, 1985), (Applegate et al., 2006), (Grabusts, 2000).

Let us define the distance matrix $D = (d_{ij}), i, j = 1,2, \dots, n,$ - distance between cities i and j. Each route can be represented as an element π of all permutations among the n cities sets. If possible route set consists of all the cyclical permutations, then in total there are $\sqrt{(n-1)!}$ such permutations. The objective function is defined as follows:

$$C(\pi) = \sum_{i=1}^{n} d_{i\pi(i)} \tag{3}$$

TSP task is to minimize the objective function in all possible permutations. If n cities are located in 2-dimensional Euclidean space and d_{ij} is Euclidean distance between cities i and j, then $C_{opt}^{(D)}$ is the shortest route for a given distance matrix D.

To use SA algorithm for such type of tasks some concepts have to be introduced. For each route we can define the neighbor as a rout set that can be reached from the current path during one transition. Such neighboring structure mechanism for the TSP is called the $k - opt$ transitions. In the simplest case - $2 - opt$ transition is based on the fact that the two cities are selected on the current route and the sequence, in which the cities between these couples were visited, is reversed (see Figure 2).

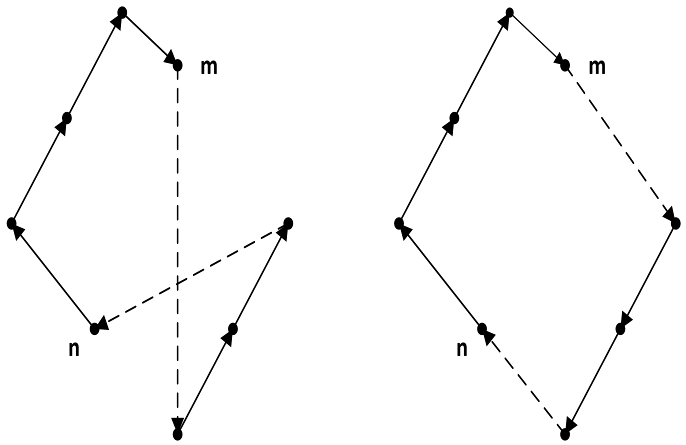

Fig.2 **2-opt example (on the left – the current route, on the right – after reversing the sequence between m and n**

Route neighbors are now defined as a set of cities that can be reached from the current rout through the $2-opt$ transitions (i.e. $\sqrt{(n-1)n}$) such neighbors).

Research part

In the research part different levels of educational institutions, educational institutions and their GPS coordinates were defined, the shortest path between educational institutions with the help of SA algorithm was computed and the attachment of educational institutions to the geographic maps was carried out.

Level 1. Kindergartens (see Table 1.)

Table 1 **Denotation and GPS coordinates of kindergartens**

No.	Name of kindergartens (in Latvian)	Latitude	Longitude
1	Čornaja *(Čornajas pirmsskolas izglītības iestāde)*	56,38478	27,415519
2	Dricāni *(Dricānu pirmsskolas izglītības iestāde)*	56,649235	27,182159
3	Gaigalava *(Gaigalavas pirmsskolas izglītības iestāde)*	56,734355	27,06622
4	Ilzeskalns *(Ilzeskalna pirmsskolas izglītības iestāde)*	56,641404	27,393226
5	Kaunata *(Kaunatas pirmsskolas izglītības iestāde „Zvaniņš”)*	56,330835	27,544932
6	Lūznava *(Lūznavas pirmsskolas izglītības iestāde „Pasaciņa”)*	56,359505	27,262984
7	Malta *(Maltas pirmsskolas izglītības iestāde)*	56,349716	27,166046
8	Mākoņkalns *(Mākoņkalna pirmsskolas izglītības iestāde)*	56,290113	27,439382
9	Nagļi *(Nagļu pirmsskolas izglītības iestāde)*	56,684951	26,928374
10	Nautrēni *(Nautrēnu pirmsskolas izglītības iestāde „Vālodzīte”)*	56,711536	27,411917

No.	Name of kindergartens (in Latvian)	Latitude	Longitude
11	Ozolaine *(Ozolaines pirmsskolasizglītības iestāde „Jāņtārpiņš”)*	56,41097	27,233186
12	Rikava *(Rikavas pirmsskolas izglītības iestāde)*	56,610504	27,033659
13	Silmala *(Silmalas pirmsskolas izglītības iestāde)*	56,396045	27,095819
14	Strūžāni *(Strūžānu pirmsskolas izglītības iestāde „Zvaniņš”)*	56,69624	27,239002
15	Uļjanova *(Uļjanovas pirmsskolas izglītības iestāde „Skudriņa”)*	56,549423	27,061344

SA algorithm in this case was carried out in 21 steps. Algorithm computed the shortest route; it was 198 km (see Figure 3). Attachment of educational institutions to the map is shown in Figure 4.

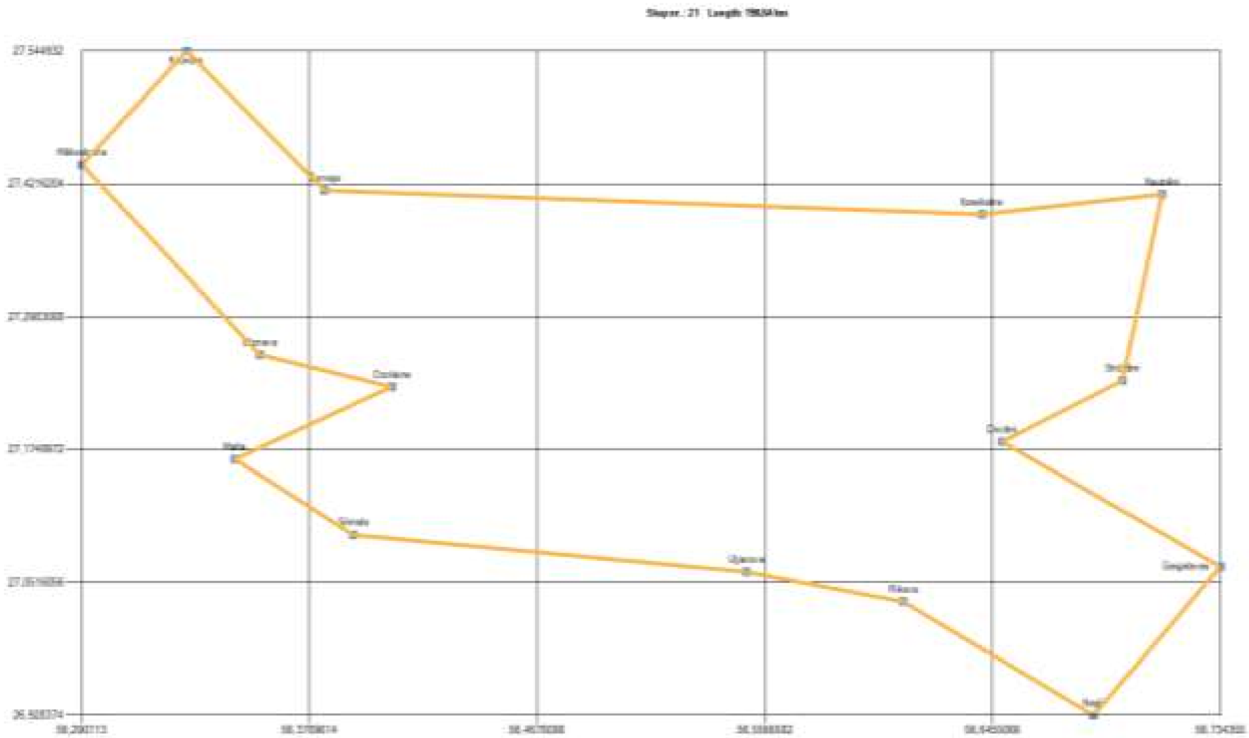

Fig.3 **The shortest path between kindergartens computed with the help of SA algorithm**

Level 2. Basic schools (see Table 2.)

SA algorithm in this case was carried out in 22 steps. The shortest path computed with algorithm was 251 km (see Figure 5). The attachment of educational institutions to the map is shown in Figure 6.

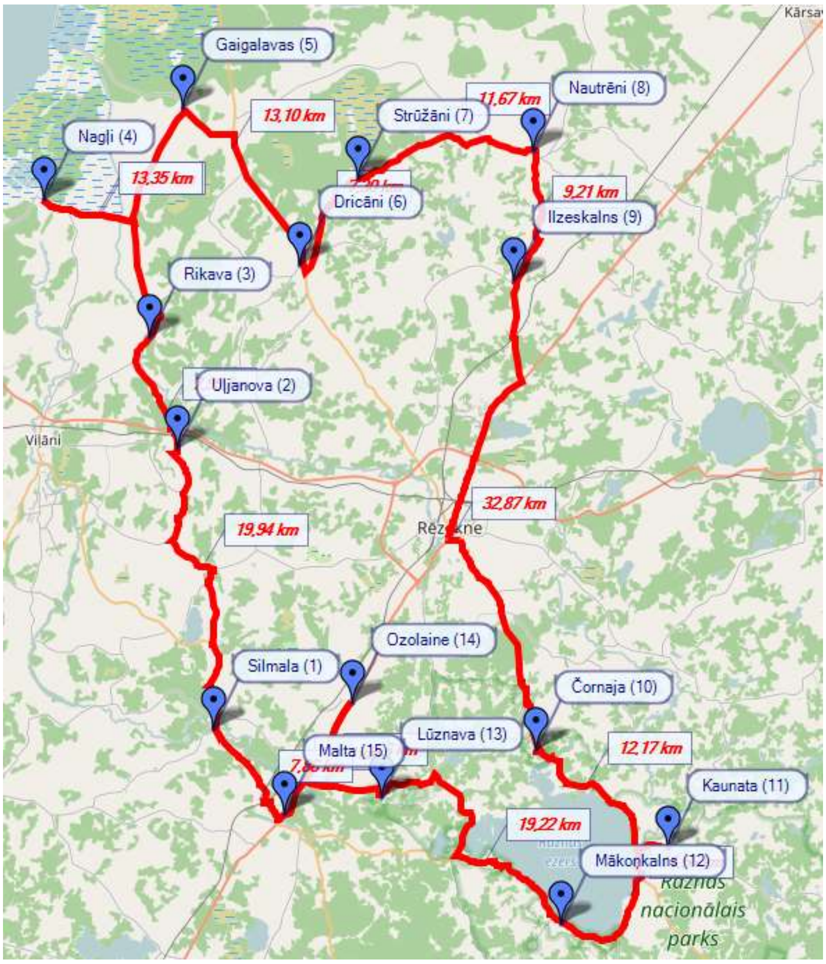

Fig.4 **The attachment of the shortest path between the kindergartens to Google maps**

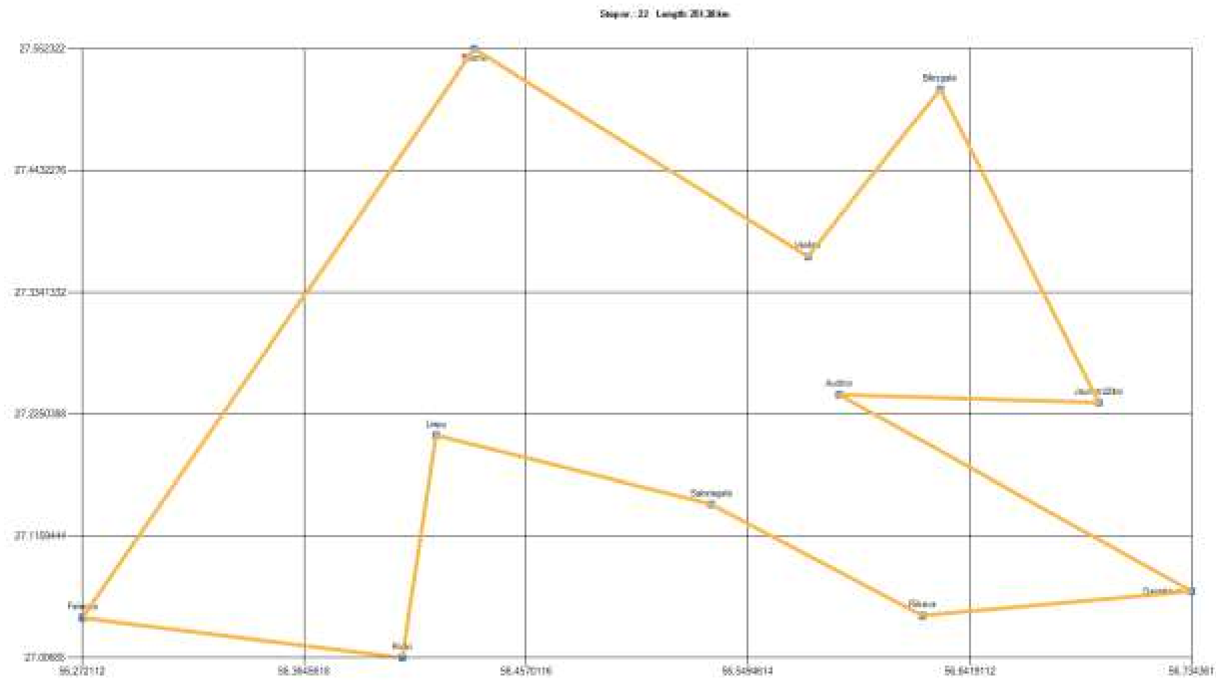

Fig.5 **The shortest path between basic schools computed with the help of SA algorithm**

Table 2 **Denotation and GPS coordinates of basic schools**

No.	Name of basic schools (in Latvian)	Latitude	Longitude
1	Audriņi *(Audriņu pamatskola)*	56,587559	27,242635
2	Bērzgale *(Bērzgales pamatskola)*	56,629493	27,516288
3	Feimaņi *(Feimaņu pamatskola)*	56,272112	27,042613
4	Gaigalava *(Gaigalavas pamatskola)*	56,734361	27,06622
5	Jaunstrūžāni *(Jaunstrūžānu pamatskola)*	56,695701	27,235483
6	Kruķi *(Kruķu pamatskola)*	56,405302	27,00685
7	Liepas *(Liepu pamatskola)*	56,419436	27,206095
8	Rēzna *(Rēznas pamatskola)*	56,435283	27,552322
9	Rikava *(Rikavas pamatskola)*	56,622145	27,044503
10	Sakstagals *(Sakstagala Jāņa Klīdzēja pamatskola)*	56,534155	27,144494
11	Verēmi *(Verēmu pamatskola)*	56,574573	27,366389

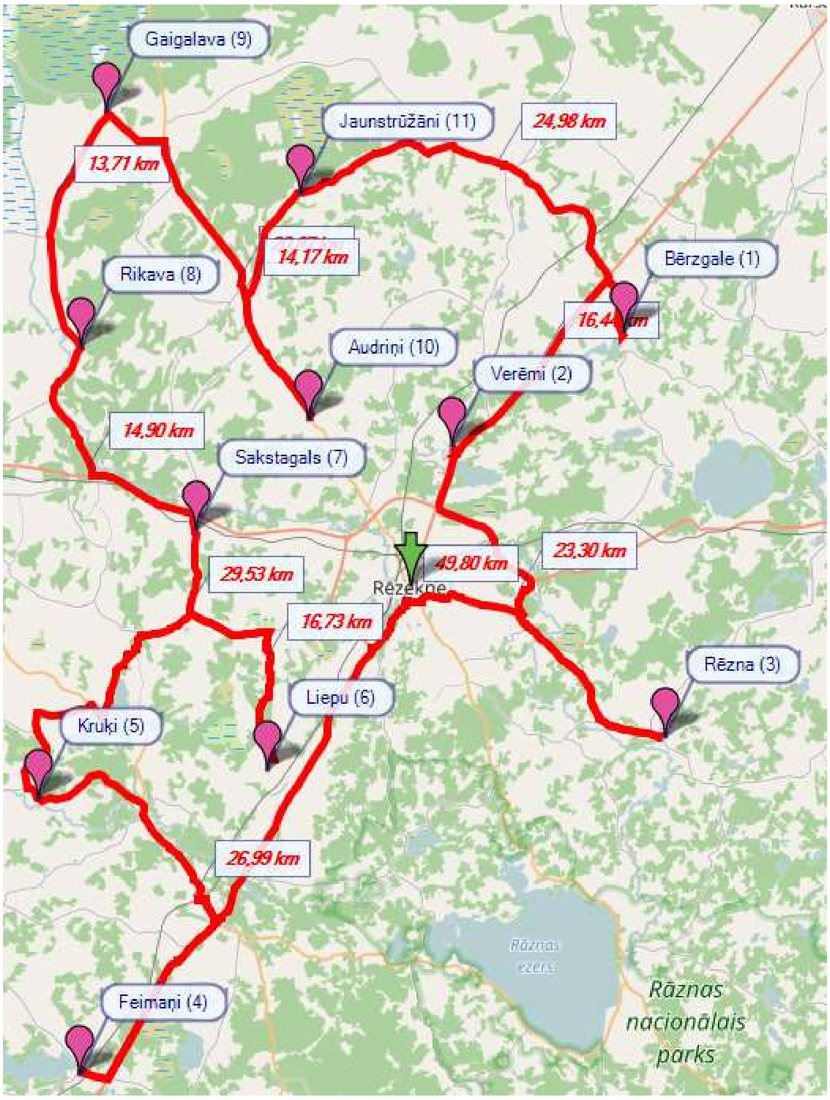

Fig.6 **The attachment of the shortest route between basic schools to Google Maps**

Level 3. Secondary schools (see Table 3.)

Table 3 **Denotation and GPS coordinates of secondary schools**

No.	Name of secondary schools (in Latvian)	Latitude	Longitude
1	Dricāni (*Dricānu vidusskola)*	56,649232	27,182524
2	Kaunata (*Kaunatas vidusskola)*	56,331737	27,543208
3	Makašāni (*Lūcijas Rancānes Makašānu Amatu vidusskola)*	56,587671	27,315964
4	Malta (*Maltas vidusskola)*	56,347054	27,157439
5	Nautrēni (*Nautrēnu vidusskola)*	56,71153	27,412196
6	Tiskādi (*Tiskādu vidusskola)*	56,405377	27,007207

SA algorithm in this case was carried out in 17 steps. The shortest path computed with algorithm was 162 km (see Figure 7). The attachment of educational institutions to the map is shown in Figure 8.

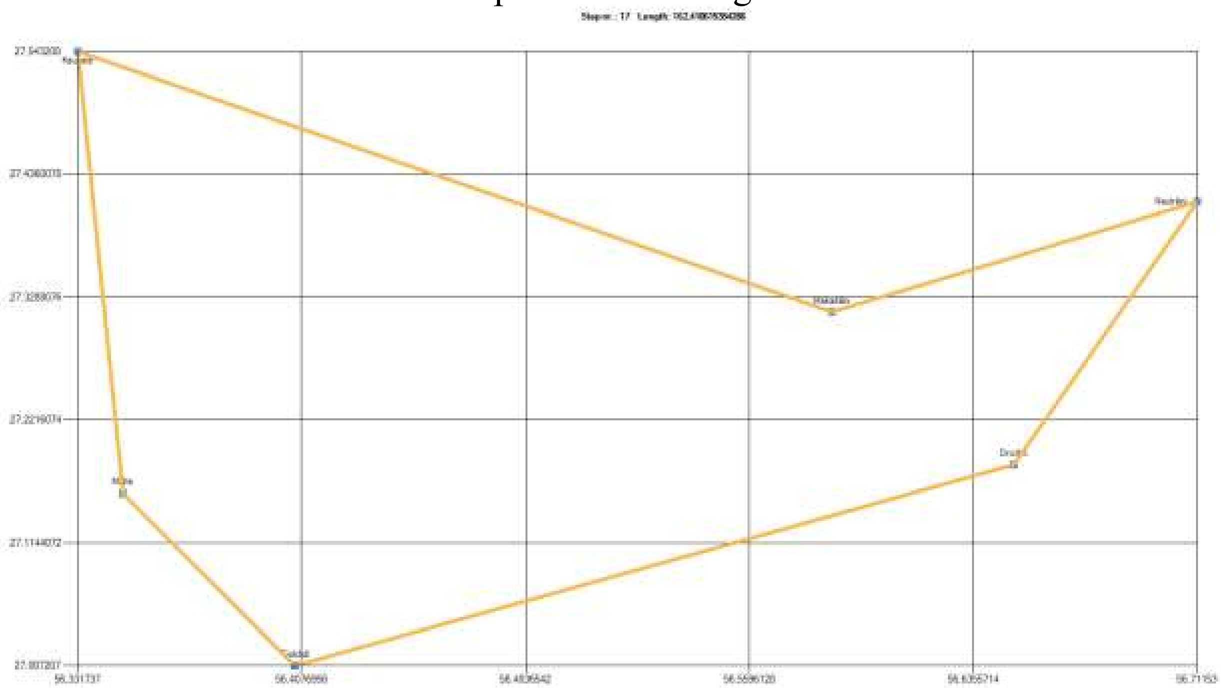

Fig.7 **The shortest path between secondary schools computed with the help of SA algorithm**

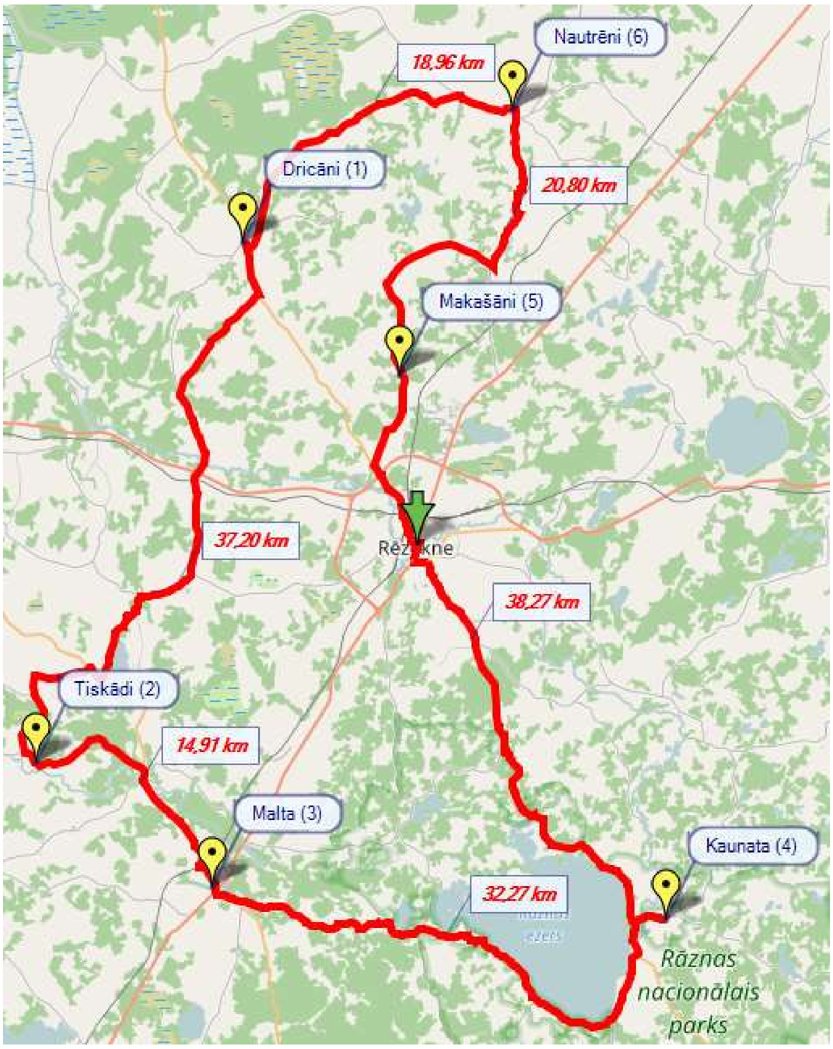

Fig.8 **The attachment of the shortest path between secondary schools to Google Maps**

Similarly, statistics on Level 4 and 5 was collected. How can it be used practically? Supposing, that there is a need to find out the optimal distance between secondary schools, basic schools and kindergartens, make the attachment of these educational institutions to the map with the purpose to analyze the potential children closeness to the educational institution. (see Figures 9 and 10).

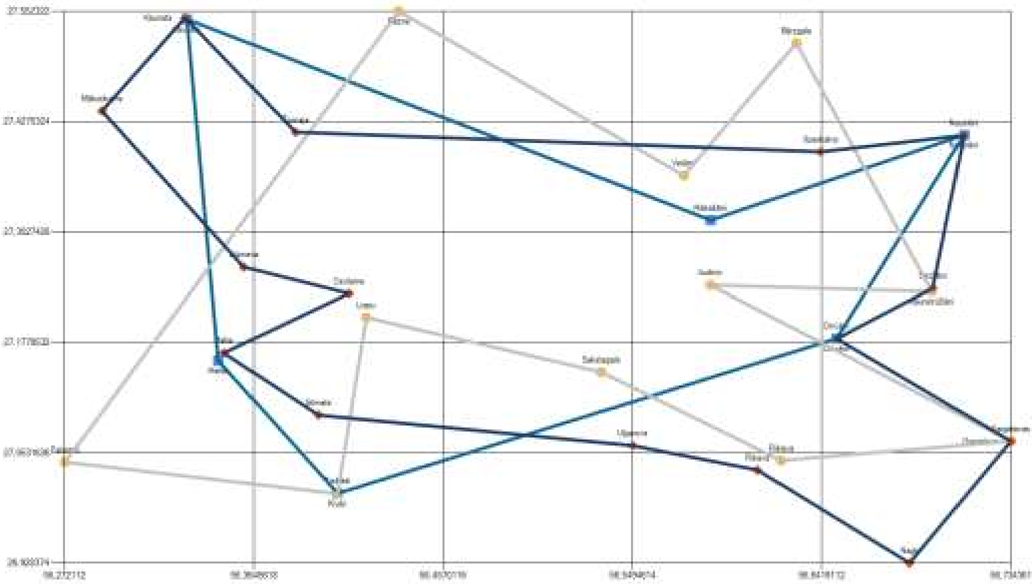

Fig.9 **The shortest computed route between the three groups of educational institutions**

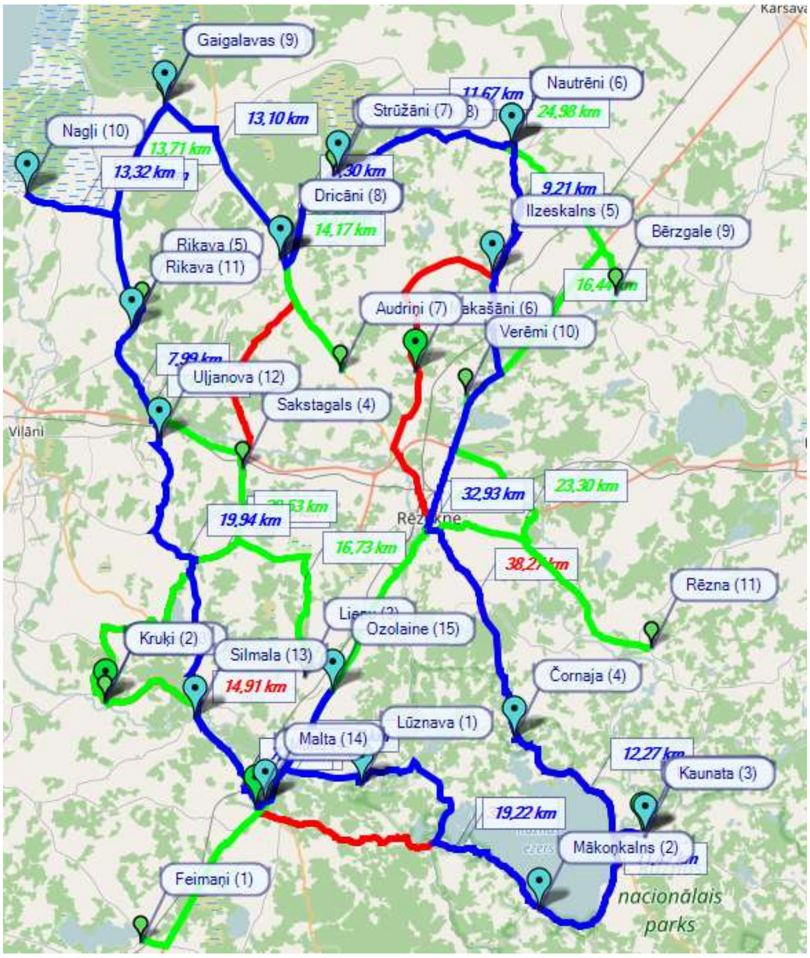

Fig.10 **The attachment of the shortest route between the three groups of educational institutions to Google Maps**

We assume that the result is trivial for the heads of educational institutions. It is clear, that for the children after finishing Čornaja kindergarten it is nearer to get education in Rēznas basic school or Kaunatas secondary school. But in our case, a theoretical modeling tool is offered, when one of the educational institutions is hypothetically excluded from "circulation".

Conclusions

We proposed that our simulation result is relatively simple, but in case it was needed to exclude a school from the existing network of educational institutions, it would allow to model overlapping of educational institutions on the map and determine children potentially shortest route to the chosen educational institution.

In this study the software that allows to find the shortest path between different educational institutions in Rēzekne Municipality with a purpose to optimize and determine the shortest path between educational institutions has been developed. A multi-tiered architecture in educational institutions characterization and overlapping is offered. The aim of the study was to develop the modeling tool for analysis of potential educational network optimization.

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