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IDENTIFYING STUDENTS' WAYS OF LEARNING OF MATHEMATICS AT UNIVERSITY LEVEL

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Abstract. *The Mathematics study course is one of the core subjects in study programs of Technical Universities. To acquire this course successfully it is necessary to have mathematics background of sufficiently high quality. The authors of this paper recognize the difficulties first year students face due of their insufficient mathematical knowledge.*
Today, universities emphasize independent study work by students and allocate special time slots for this. To be successful, students need to plan their study time, use appropriate learning methods, and have motivation. Because of the significance of students' individual work, a questionnaire was developed to research how students plan their time and activities for learning mathematics.
The authors selected three focus groups of first year students at Riga Technical University (RTU), Latvian Maritime Academy (LMA), and University of Latvia (UL) to collect the data. The comparative analysis of data showed how students use the time slots allocated by institutions. The UL and RTU students on average do not fulfil this time completely, while the LMA students spend more time for learning mathematics. Students highly value individual consultations with teachers; they actively communicate with study mates to solve homework assignments; and students use information technologies in the study process.
Keywords: *learning at university level, mathematical knowledge, time, transition.*

Introduction

The Mathematics study course becomes an increasingly important service subject for a range of disciplines - not only as a background subject in science, technology, engineering, and mathematics, it plays a significant role in economy, sociology, agriculture and in other fields. To reach success in studies and to become highly skilled specialists, students need a sufficiently deep level of mathematical knowledge. However, the transition from secondary school to the university are challenging for most students. To overcome the difficulties faced by many students, it is necessary to organize the learning process. We, the authors

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of the presented paper, are responsible university teachers, and we look for innovative teaching methods to initiate and support students' learning to enhance their level of expertise in the subject of mathematics.

There is the problem of the wide diversity in the *level of mathematical knowledge* (LMK) that students bring from the high school to the university. The commission of *centralized exam in mathematics* (CEM) of Latvia decided to eliminate the level of demands to the score of 5% of correctly solved problems on the exam as an accepted positive result. It is expected that graduates with low scores in mathematics exam do not choose to study specialties that are strongly based on mathematics. Nevertheless, some of them started studies in specialties that need valuable mathematics background.

This diversity of assessed results on CEM creates difficulties both for students and for university teachers. For example, a fifth of the first-year students at LMA had CEM assessment between 5% and 40 %. The acquiring of every new theme introducing new objects, objects' properties and relations, notions and terms, and particular methods to solve the tasks and to prove the propositions requires serious effort for students with weak mathematical abilities. The teachers must apply a differentiated approach in the management of study work. The question arises here: How to make the mathematics course accessible for any student, how to strengthen and improve his or her mathematical knowledge and ability? Taking into account the significance and effectiveness of students' learning styles, we paid a closer look to the time division devoted to mathematics. The research question that we investigated is:

- *How do the students plan their time for learning mathematics and how do they value the benefits gained from different learning activities?*

The challenges in learning mathematics in the first year of university

The transition from secondary school to university essentially changes the students' learning experience. They can face different challenges: they are confused by the increasing speed of incoming information, they get more autonomy given for the solving of tasks, they have to grasp the sense of the abstract methods of proofs. Artigue (2016) points to the discontinuities between algebra and analysis that have been proved challenging for students – to understand the mechanism of analytic proofs, to solve equalities and inequalities as part of analytic calculations.

Many researchers speak about the amount of problems and questions derived from the students' start of studies in the first year of university. One of the widely known problems is the students' insufficiently strong mathematics background knowledge. Survey Team 4 from 12th International Congress on Mathematics Education (Thomas et al., 2015) reported about the internationally performed survey to obtain data on the above-mentioned transition. They collected data answered by academic mathematics teachers from 21 countries. 91.1% of respondents agreed that students have problems in moving from school to university, and they commented that difficulties come from lack of preparation in high school; from differences of the teaching style and of the theoretical content. Other researchers (Hoyles et al., 2001) pointed to the students' lack of essential technical facility in algebraic manipulations. They noted that students do not understand that mathematics is a precise discipline in which exact, reliable calculation, logical exposition and proof play essential roles. Rensaa and Grevholm (2017) argued that students have difficulties with proofs and formal mathematical language that appears in studies without connection to previous mathematics knowledge.

Several researchers discuss the challenge: how to bridge the gap in the transition from high school to university because of the difference in students' level of mathematical knowledge. Australian researchers (Nicholas et al., 2015) present their case of secondary education curriculum - senior grade students can choose to learn elementary, intermediate or advanced mathematics or can choose to not learn it at all. Students with a different mathematics background or without one enter universities to study disciplines where the mathematics knowledge is a necessary prerequisite to take particular specialization courses.

Another type of difficulties faced by university lecturers is the diversity of the audience that demonstrates a broad spectrum of abilities and levels of interest in mathematics (Wiggins et al., 2017). The attempt to keep the learning environment suited for any student is challenging. It may happen that academically strong students feel bored, or students with lower mathematical ability cannot follow the topic.

The motivation to learn mathematics to a deeper extent is not recognized by engineering students; the links between mathematics and engineering are not made explicit. Harris et al. (2014) note that in the job market there is the need for professional engineers to think mathematically and to use mathematics to describe and analyze different aspects of the real word. Interviews with the students reveal that they do not know how mathematics will be useful in their engineering courses. Lithner (2011) pointed out that several students tried to use mathematical formulas mechanically without the understanding of their content and sense.

Based on our experience, we recognize similar problems that arise in mathematics education at the university level in Latvia.

The view on learning process

The purpose of learning is to achieve defined learning goals. The components of this process should be investigated based on different theories.

Bloom et al. (Bloom et al., 1956) observed three main domains that cover the learning objective; these are cognitive (knowledge-based), affective (emotions-based), and sensory (activity-based) domains. The cognitive domain contains knowledge as the basis for development of intellectual skills. The structure of this domain was revisited after some years by a group of researchers now calling categories of these skills in the hierarchy of their complexity: *Remembering, Understanding, Applying, Analyzing,* and *Creating* (Anderson et al., 2000). The teaching and learning in 21st century names many educational theories in the context of psychology, philosophy, and pedagogy – for example, Richard Millwood's exhaustive visual graphic summarizes 32 learning theories (Heick, 2019).

In our research we focus on the activities carried out in the learning process. The process is managed and guided by academic study programs, timetables, demands stated in course descriptions and assessment of students' regular work and their achievements. The learning process is supported by lectures, practices, study materials, workplaces, and computers. Learning process is performed by a person acting individually or in communication with other persons and it contains a set of components depending on individual qualities and external circumstances (Fig. 1).

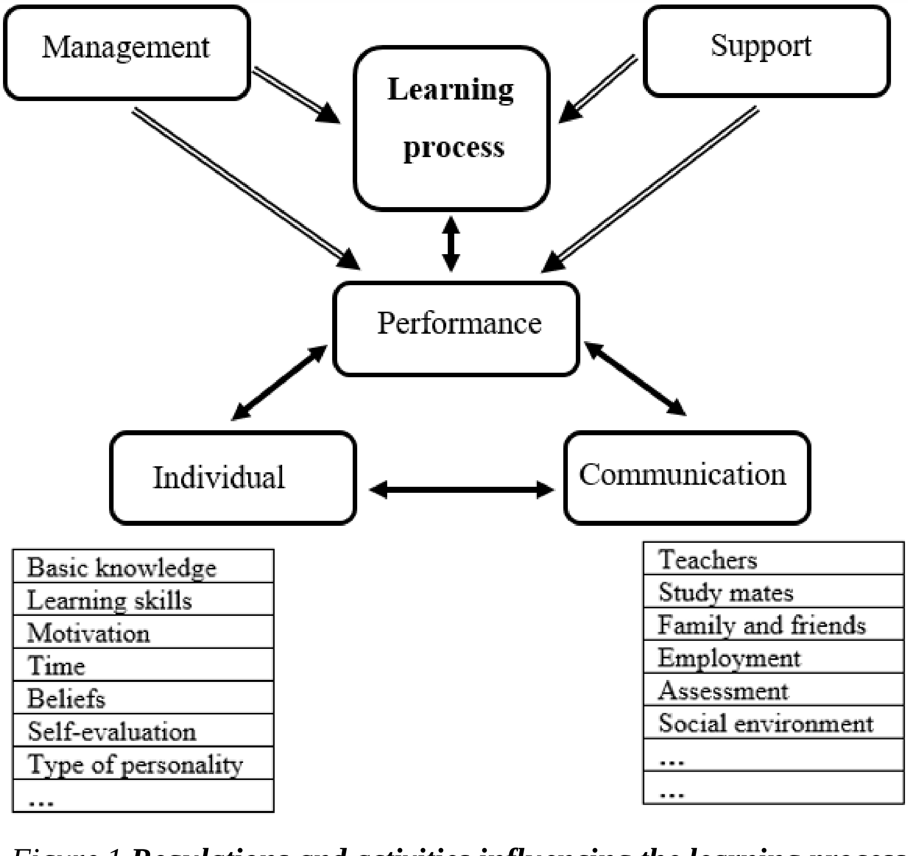

Figure 1 ***Regulations and activities influencing the learning process***

Basic mathematical knowledge and learning skills are the crucial conditions for effective learning. Accascina et al. (2014) note that often students do not recognize what they do not know. So, identifying and filling gaps in the knowledge requires additional effort and energy, where the assistance of a competent consultant is required. The role of the consultant should be taken by a university teacher, private teacher or study mate. The quality of learning depends on several aspects of motivation: is the student interested in mathematics? is he or she aware of mathematics applications in particular study subjects? Or, motivation can be quite pragmatic – based on the necessity to pass the mathematics exam. The relevant factor in the study process is time that can be valued from three positions: time as an imaginary factor (planned by the university institution to acquire the mathematics course); time as a subjective factor (time that is needed for an individual to accumulate the knowledge); time as an objective factor (time that an individual can really spend on learning depending on outer circumstances). No less important is the division of the time devoted for learning: studying of theoretical questions, solving tasks, using computer technologies and interactive materials, communicating.

Accumulation of mathematical knowledge

Mathematical knowledge of an individual can be considered as a complex, hierarchical system. Rensaa and Grevholm (2017) describe conceptual and procedural knowledge related to learning. They refer to the model introduced by Hebert and Lefevre who defined the conceptual knowledge as connected networks of knowledge that are rich in relationships. The procedural knowledge is defined as part of symbolic representations that are used by the other party to construct step-by-step solutions.

We have a similar view on the architecture of mathematical knowledge. This system contains the *storage of information units* (domain of facts, concepts, and methods) that are managed by *supervisors* – the procedures that organize, set in order, translate, form the interconnections between the units, and construct individual concepts. Supervisors create a live, flexible and coherent network. Procedural knowledge contains a set of *instructions* on the selection of information units and on procedural steps and methods to operate with these units. These instructions construct the solutions of mathematical tasks and problems, and construct the proofs. The completeness and coherence of the knowledge system and the meaningful application of procedural steps characterize the depth of mathematical knowledge. Every new concept should be accumulated and incorporated in this system by the implementation of acquired knowledge.

Incorporation of new concepts and objects increases the depth of knowledge. If the store of information units and procedural steps is incomplete, fragmented and weakly connected, an attempt to collect new information can even disarrange the balance between units and procedures. The assistance of a competent expert is proven in helping a student to improve and to arrange their system of mathematical knowledge. A group of researchers (Barzel et al., 2013) report on their experience designing special tasks and instructions to organize knowledge. These tasks and methods can be involved directly in the teaching-learning process so that students can accumulate well-organized information.

Investigation of the ways to learn mathematics chosen by students

Recognizing the significant role of learning activities in the process of forming the mathematical knowledge, we carried out a local investigation about students' ways to learn mathematics at university level. We chose three focus groups of students in the first study year - 8 students of the Mathematics specialization and 13 students of the Teachers of Mathematics specialization from the *University of Latvia* (UL); 53 students of Technology of Chemistry specialization from *Riga Technical University* (RTU); and 67 students of specialization Navigation from *Latvian Maritime Academy* (LMA). The students of RTU and LMA study mathematics as a service subject. They are introduced to the concepts that are used to form notions and to solve problems in particular study subjects in their specialization. The Mathematics course includes calculus that is broadened by pre-calculus content. The students of UL study Calculus as one of the study subjects.

Institutions allot a different number of credit points to these study courses and prescribe different time slots for independent work. Prospective mathematicians have to spend 9 hours per week for individual learning, while prospective mathematics teachers and students of technology of chemistry - 6 hours per week, whereas the students of navigation spend 3 hours per week.

We composed a questionnaire to collect the characteristics of division of time devoted to learning mathematics and students' beliefs on which ways can give them more valuable benefits. The questionnaire included questions of how they value the time spent learning alone, in a group, in consultations by a university teacher or by a private teacher, and in the classes, and what benefit they have from these activities. Special attention was paid to individual learning (learning alone): how students value their reading of mathematics theory books, using of notes made by themselves, doing homework, solving additional exercises to better understand the themes, using computer technologies. Another question was how many times per week they need for learning mathematics.

We separated students in 6 groups in accordance with their CEM assessment. In the first group we included the students that have score less than 41%, in the next groups – students with scores between 51% and 60%, 61% and 70%, 71%

and 80%, and in the last group were included students with the highest score of above 80%.

Results

From the collected answers to the questionnaire we found that students of technical universities valued the benefit of the ways to learn quite similarly. Science students (prospective mathematicians and mathematics teachers) are academically more strongly prepared for studies - they do not need private lessons and they mostly learn independently. In comparison, the students of UL have higher CEM scores. The mean CEM score for UL students is 75.7%; for students of RTU – 62%, and for students of LMA – 54.2% in the selected focus groups.

Figure 2 demonstrates the mean value of the benefit gained at a particular activity as estimated by students.

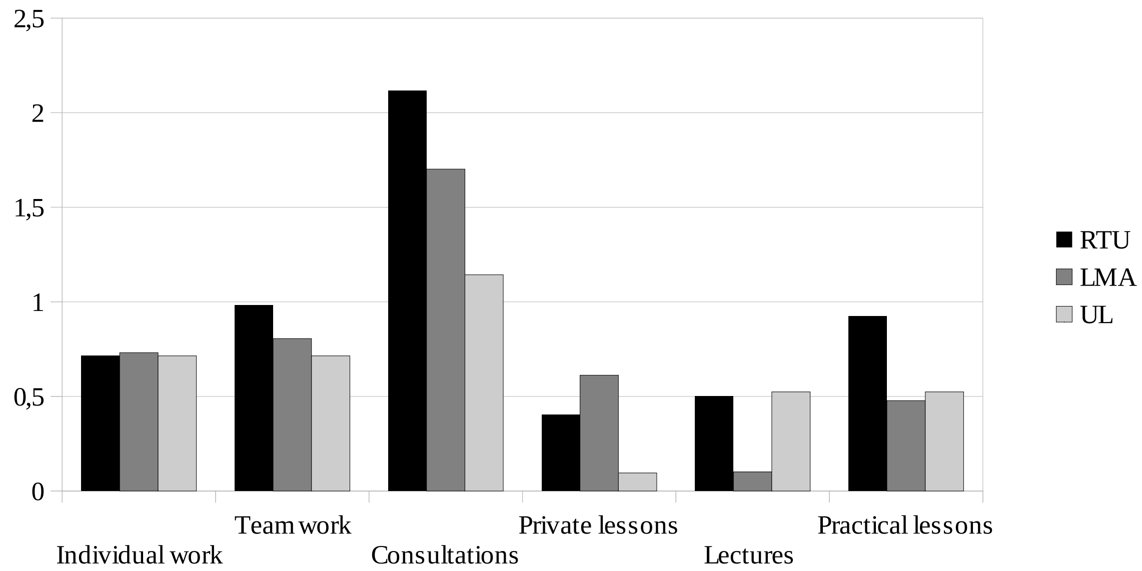

Figure 2 ***Students' opinions on the benefits of different ways to learn***

The students from all institutions answered that the best way to learn is a consultation with their university teacher. As the next relevant activity is rated teamwork with their study mates and private lessons for those students who take them. There is minimal benefit from lectures in large audiences, especially for students with minimal mathematics background. For instance, the case of LMA students presents direct correlation between the students' level of mathematical knowledge and their estimation of the benefit from lectures. The half of them with a moderate or low CEM score are hindered to engage in the classes more actively.

Students with a strong prior mathematics background accept the benefit of the auditorium work.

Speaking of the time that students devote on acquiring all study subjects, we found that on average they do not learn as much as planned by the institution (20 hours per week for all study subjects). Still, part of the focus group students dedicate more energy for studies than others - 11% of RTU students, 24% of UL students, and 25% of LMA students learn at least 20 hours per week.

Figure 3 demonstrates the data on the average time spent for learning of mathematics (Fig. 3). The students of UL use less time than planned to acquire the topics of Calculus. The need for more time is evident for students with lower level of mathematical knowledge; especially the students of Navigation specialization need twice more time than it is planned. Here we would like to note that one of the Technology of Chemistry students is very persistent, using 30 hours per week for mathematics learning.

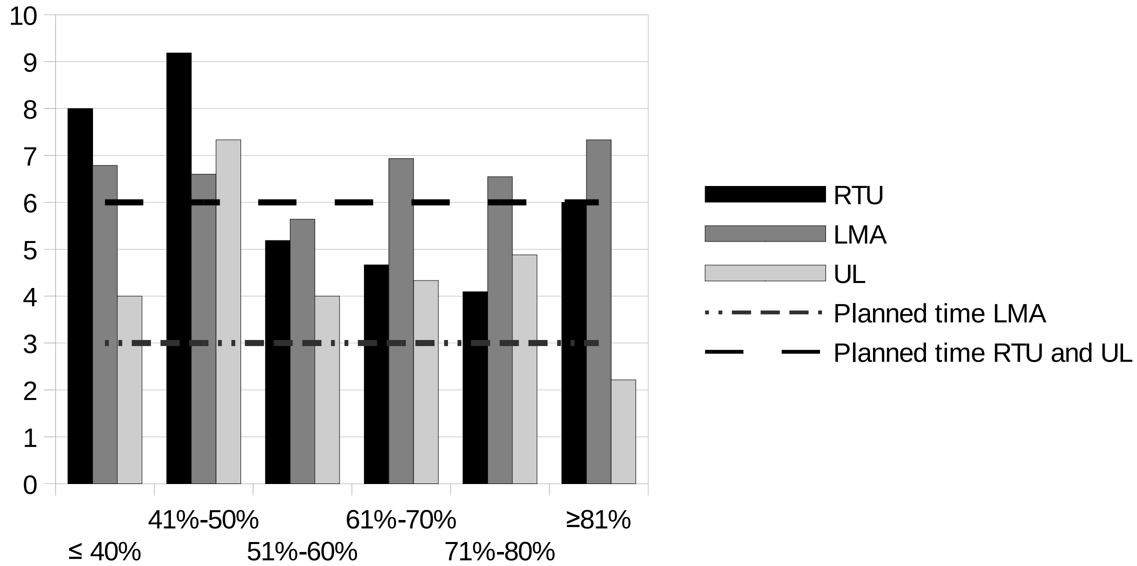

Figure 3 ***Time spent for after-class learning of mathematics***

We analyzed the ways of individual learning as well. Students of all institutions show approximately the same preferences. Mostly they learn from the notes made by themselves to read theory and to find examples of the task solution which is expedient to prepare their homework. Up to 40% of the learning time for independent work is used to apply the advantages offered by computer-aided technologies - demonstrations of the examples of task solutions, explanations of theoretical terms, applications of interactive study materials, or computer programs for solving mathematical problems.

The reading of mathematics textbooks and the solving of additional problem sets are not broadly used. Such approach can influence the quality of knowledge, because note books may not contain complete information.

The questionnaire also asks some qualitative questions about the learning process: how do the students evaluate the mathematical knowledge acquired during the semester, and do they learn enough? We found that the students' self-estimation was highly similar to their score on the centralized exam. Only a few students from all focus groups agreed that they work hard, whereas the others feel that their attempt to get quality knowledge is not sufficient. We have to note that engineering students complain about the too-high speed in the introduction of new concepts of mathematics, and they desire to solve easier tasks.

Discussion: Possible activities to improve students' learning

We found from the collected data that students with low background in mathematics have the predicted difficulties in acquiring mathematics course at university level. Academic teachers face the challenge: how to help students to improve their knowledge? One of the aspects is to organize the presentation of the lecture including more examples, challenging tasks on different level of difficulties, causing discussions, engaging the students in investigation activities. Another way is to recommend effective learning methods to students.

There have been various studies on the effectiveness of the lecture as a teaching method (Charlton, 2006; Schmidt et al., 2015). The researchers discuss advantages and disadvantages of the lecture teaching method. The following advantages are mentioned: with this teaching method, a large number of topics can be covered in a single class period; learning material is not required; students' listening skills are developed. Charlton (2006) researched the communication between the auditorium and the lecturer, and stresses the necessity of trust between students and the lecturer as a condition of effective learning. Several disadvantages can arise with this teaching method: for example, the language used in the lecture is above the standard of the students, so the students are not able to grasp the theme of the lecture and understand its content; the students cannot to develop critical thinking.

In practice, it has been verified that students give positive feedback if the following elements are added to lectures:

- at the beginning of the lecture the learning objectives are formulated;
- regular tests using the opportunities offered by IT are included in lectures: students take short (5-10 min) tests on their mobile devices about previous or current lecture topics in order to check student progress, also for the diagnosis of previous knowledge. The test can be prepared in e.g. *socrative.com, kahoot.com, mentimeter.com*. If tests are

performed electronically, it is immediately possible to see the students' mistakes and discuss them;

- to ensure that students are actively involved in the lecture, the lecturer can ask a question and put the students to answer on their mobile devices; use a show of hands to check the responses; if incorrect answers are chosen teacher can use the opportunity to talk the mistakes through with students;
- different learning methods should be used to ensure that students do not lose interest in actively participating in lectures. Students like lectures in which they need to think actively and work actively; they like lectures that are "*outside the box*" (i.e., "outside" just to listen and sometimes write down notes if necessary during the lecture);
- various visual demonstrations should be used, different examples and applications, historical facts, and activities that can involve students.

Several researchers turn their attention to the influence of textbooks in the learning process, to the organization of knowledge, or to the quality of e-learning environment. The researchers Rensaa and Grevholm (2017) acknowledge that visualizations may play an important role in learning mathematics. The pictures included in text books to illustrate examples are highly evaluated by the students. Grevholm (2005) discusses a different approach in the construction of mathematical comprehension. She proposes the construction of concept maps that include various properties of objects noted as nodes of the map and edges showing different types of interconnections of objects. Such interpretation is useful to detect students' misinterpretations by comparison with the expected results, and can help them develop better understanding about the relations of mathematical concepts.

Pan and Hawryszkiewycz (2004) categorize processes in Web-based learning environment. Researchers state that many instructional websites deliver course materials without the guidance for effective use of these materials. They discuss the architecture of such learning services in supporting the formation of constructive knowledge to assist learners in building new competences.

Following the findings of the education science, academic teachers should be open for discussions with their students – not only to give exhaustive explanations but to ask meaningful questions that can guide students to search for the solution of the problem themselves.

Conclusions

Comparative analysis of collected data from questionnaires points to several tendencies worth contemplating:

- Students prefer consultations with teachers, private consultations, and to study mathematics with mates. This indicates that it should be more effective to organize the study process of a mathematics course in groups smaller than those used today.
- A significant part of students assert that their benefit from the lectures is rather low. Most students complain about the speed and amount of information. Nevertheless, the quality of lectures is accepted by students with higher CME scores. The question arises whether it would be rational to give separate lectures for students with different levels of knowledge.
- Students plan learning activities in a rather balanced way and they apply information technologies in their independent study work. The authors note that such experience enriches students' mathematical knowledge.
- The time that students spend studying mathematics differs in focus groups. Students with lower CME score spent more time learning. The LMA students spent more time on average than the RTU and UL students. Here arises the question: how effectively do the students learn? This question is due to the students' opinion that they learn not enough. Further investigation of students' learning methods is needed.

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