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PARAMETERS IN FORMULATIONS AND SOLUTIONS OF INTRODUCTORY PROBABILITY PROBLEMS

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Abstract. *Mathematical problems with parameters offer a higher semiotic complexity level of mathematical activities. The topicality of the research is determined by the fact that there are no studies on types of parameters in formulations and solutions of probability problems. The study aims are to analyse the current literature and propose an approach to classify parameters depending on their nature. Methodology - qualitative content analysis of probability problems from published textbooks and research papers. The main result - a parameter classification and interpretation scheme for introductory probability problems. The proposed parameter classification can help differentiate and individualise the study of probability theory and statistics.*
Keywords*: combinatorics, higher education, probability theory, problems with parameters, school education.*

Introduction

A mathematical problem becomes more challenging when parameters are introduced because the inclusion of one or more parameters implies a higher degree of algebraization and a higher semiotic complexity level for mathematical activities (Drijvers, 2003; Godino, Neto, Wilhelmi, Ake, & Etchegaray, 2015; Sedivy, 1976). Parameters act as meta-variables and have a hierarchically higher position compared to variables in mathematics (Drijvers, 2003).

The article aim is to analyze the current literature and propose an approach to interpretation and classification of parameters depending on their nature in introductory probability problems.

The research method is a qualitative content analysis of probability problems from published textbooks and research papers.

In general, problems with parameters are commonly used in mathematics courses. A common type of problem with parameters is equations and systems of equations. One can mention linear and polynomial systems over real, integer, or finite fields. The introduction of parameters in combinatorics problems corresponds to the possibility of generalizing these problems for any number of elements (Krastina, Sondore, & Drelinga, 2015). In courses on linear algebra, one finds problems involving matrix operations, the rank of matrices, and the

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computation of determinants as a function of parameters. Number theory is an area where problem complexity and the nature of the solution depend strongly on the parameters. For example, one can mention problems related to integer factorization. In polynomial algebra, there are problems related to finding roots and factorization of polynomials whose coefficients depend on parameters. A considerable number of problems in probability theory used in school and college courses are problems with given parameters. Moreover, a probability problem without parameters in its formulation can be easily transformed into a *probability problem with parameters* (PPP). Thus, one can ask the question about possible types of parameters in probability problems.

The main result of the research- the authors propose to consider primary and secondary parameters, each of which can be divided into two classes.This work is a continuation of the study on identifying different types of parameters for combinatorics problems (Sondore & Daugulis, 2018). Several examples (some examples were created by the authors) are analyzed in this article to illustrate the classification presented by the authors. The correspondence of these examples to other published classifications is determined. First, PPP's are compared with a classification of probability problems according to the six levels of algebraic reasoning given in (Burgos, Batanero, & Godino, 2022). Second, it is determined how the four levels of understanding and interpretation of the concept of parameters in algebra (Drijvers, 2003) correspond to parameters by authors' classification.

The research is relevant because no articles analyzing the role of parameters in PPP are accessible to college professors and teachers (the target audience of this work). The parameter classification proposed by the authors indicates ways to select problems for both generalization and individualization of the university study of probability theory and statistics.

Literature review

The parameters in a mathematical problem are denoted by letters. The use of letters is a fundamental step on the way from arithmetic to algebra, as pointed out by (Furinghetti & Paola, 1994). The use and meaning of letter symbols (signs, something that denotes something else) is one of the basic problems in learning algebra since letters and numbers have different roles in the algebraic context (Bardini, Radford, & Sabena, 2005; Heck, 2001). Studies on the use of parameters mostly analyze the main methods for solving problems with parameters and the effects of changing parameters for some families of functions, for solving equations and inequalities with parameters. Students' misunderstandings of using parameters in these objects affect their learning of mathematics (Bardini et al., 2005; Chow, 2011; Drijvers, 2003; Godino et al., 2015; Sedivy, 1976).

Although there are no articles that directly analyze the classification of parameters in probability problems, there are classifications that can be related to it. Probability problems have been studied by authors (Burgos et al., 2022) to classify tasks according to algebraic levels of reasoning - from proto-algebraic levels of mathematical activity to higher levels of algebraization and formalization. A description of these levels according to (Burgos et al., 2022) is as follows. Proto-algebraic level 1 is characterized by the introduction of some simple algebraic objects or processes. At proto-algebraic level 2, probabilities are calculated and the simple inverse proportional equation is formulated and solved. At the strictly algebraic level 3, in addition to these processes, systems of equations are set up symbolically and the linear equations are solved by substitution. Level 4 is characterized by the first appearance of parameters in the determination of probability, level 5 - by operations with parameters and statistical inference, but level 6 - by working with algebraic structures - operations with sets and with probability functions. PPP's correspond at least to the fourth level of algebraic reasoning. Looking more closely at the nature of the parameter, one finds a description of four levels of understanding and interpretation of the concept of the parameter in algebra - the parameter as a placeholder, changing quantity, generalizer, and unknown, but the role of the parameter can change during the problem-solving process (Drijvers, 2003). The authors concluded that the concept of a parameter in PPP corresponds to all classes of this classification. Since each possible parameter value defines a specific, simpler problem, the parameter in PPP is a placeholder. The parameter as a changing quantity in PPP means that the solution formula changes significantly. The parameter as a generalizer in PPP means that it is necessary to obtain a general parametric solution with a reification of the formulae. Consider the following example: find parameter values for which the probability of an event is 0. In this case, the parameter is unknown.

Methodology

In probability theory, parameters are used to describe distributions such as the binomial or normal distribution. In this paper, the authors use parameters in a different application sense - this study is concerned with parameters in formulations and solutions of introductory probability problems. These problems are concerned only with procedures such as: finding the number of combinations without repetitions; calculating simple and composite probabilities with the product rule or the sum rule.

Remark. The total number of subsets of n distinct objects taken k at a time can be calculated by combinations C_n^k. The notation of combinations is taken from the standard learning materials in Latvia, see (Krikis & Steiners, 2001; Smotrovs, 2004; Uzdevumi.lv, 2022), where the parameters n and k are non-negative integers $n \geq k$. Therefore, it must be taken into account that, for example, the value C_6^7 is not defined. Although one could extend the bounds on the parameters n and k to define the number of subsets by $C_n^k = 0$ if $n < k$.

The authors conducted a qualitative content analysis of introductory probability tasks. A number of these tasks was selected and then solved. Solutions were investigated if they were given. The problems studied correspond to the level of knowledge of the last year of secondary school, but mainly to the level of university probability theory and statistics. Formulations and solutions of probabilistic problems were studied in published textbooks and research papers (Aigner, 2007; Anderson, 2004; Andreescu & Feng, 2004; Conroy, 2018; Gusak, & Brichikova, 2002; Krikis & Steiners, 2001; Krikis, Zarins, & Ziobrovskis, 1996; Meshalkin, 1973; Ross, 2010; Roussas, 2007; Smotrovs, 2004; Steiners, 2001). Other resources were also analysed, for example (Uzdevumi.lv, 2022) and an online calculator for dice probabilities (Sas, 2021).

Parameters in formulations and solutions of introductory probability problems are classified according to the type of parameter. By this, the authors mean the following aspects of the problems:

- presence or absence of explicit parameters in the problem formulation;
- introduction goals for parameters, if they are introduced in the solution process;
- description of the parameters and their ranges of values.

A parameter classification scheme

The authors propose to consider two classes of parameters for introductory PPP, each of which can be further subdivided into two subclasses. If the probabilistic problem already has a lettered parameter in its formulation, the authors suggest that it be called the *primary* parameter. A parameter is called *secondary* if it is introduced in the problem-solving process.

It is noted that there are two types of primary parameters depending on whether the parameter specifies the number of elements or the probability of an event. An *enumerative primary parameter* is a variable that takes values in a subset of the non-negative integer set N_0. Accordingly, a *probabilistic primary parameter* is a variable that takes values in the interval [0;1]. Secondary parameters can be divided into two classes, depending on the purpose of their introduction. If several subcases have to be considered in the solution, the answers for these cases are very different and no simple single formula is possible, it is necessary to code these subcases with *hidden parameters*. If the purpose of introducing a parameter is to facilitate the process of solving the problem, such a parameter can be called an *auxiliary parameter*.

In the following, the authors give examples with different parameters. The research results (Sondore & Krastiņa, 2018) indicate that students have difficulties solving combinatorial problems related to real situations. Therefore, the problems included are related to experiments and games. Note that in any introductory PPP, the challenge is to identify special cases (extreme points) in the parameter domain and divide them into sub-domains so that all parameter values for a given sub-domain have the same solution formula.

Ex.1. gives an experience of interval splitting for the primary parameter domain and the need to check the answers to the solution given in the textbook. PPP from Ex.1. and Ex. 2 can be used for constructing multiple individualised tasks using different specific parameter values.

Ex.1. There are n tickets in a lottery, of which m are winners. How large is the probability of a win for a person holding k tickets? The answer is $1 - \frac{C_{n-m}^{k}}{C_n^k}$ (Meshalkin, 1973).

The textbook answer is without specifying conditions for the enumerative primary parameters *n*, *m*, *k*. The solvers themselves must recognise possible values of the parameters. *n*, *m*, and *k* are non-negative integers and $n \geq k, n \geq m$. However, the review showed that not all conditions were found. The given answer $1 - \frac{C_6^7}{C_{10}^7}$ is false for values *n*=10, *m*=4, *k*=7. This numerical test makes it possible to find the not-so-obvious condition $k \leq n - m$, the number *k* of tickets cannot be greater than the number of tickets without winning. The correct answer to Ex.1 can be found in Table 1. Ex.1 corresponds to algebraic reasoning level 4. The roles of the parameters *n*, *m*, and *k* can change during the solution process. *n*, *m*, and *k* are placeholders when the probability for a certain number of lots (number of tickets) is found; they are generalizers - when the general solution is constructed. The parameter *k* is unknown, but *n* and *m* are placeholders at the same time when the solver asks for which values of *k* the probability of winning is greater than 0.5. The parameters are changing quantity if, for example, the solver realises that for values *n*=10, *m*=4, *k*=7 the formula given in the textbook is wrong.

Table 1 **The answer of Ex.1** *(created by the authors)*

Global condition $n \in N_0$, $m \in N_0$, $k \in N_0$; *interval splitting for k and m*	the probability
$k \leq n - m; n \geq m$	$1 - \frac{C_{n-m}^{k}}{C_n^k}$
$n \geq k > n - m; n \geq m$	1

Ex.2. A password is any 10-digit number. What is the probability that a digit k occurs exactly m times in a password?

There are two enumerative primary parameters k and m. The global condition for k: $k \in N_0$, but the possible values of the parameter k are distinguished into two subdomains - k is zero and k is non-zero ($0 < k \leq 9$). After checking the answer for possible values of parameter m, the range of m is divided into three intervals. The global conditions and sub-domains for the parameters and the answer with brief explanations can be found in Table 2. For all other cases, the answer is 0. Ex. 2 corresponds to algebraic reasoning level 4.

Table 2 ***The answer of Ex.2*** *(created by the authors)*

$k,m \in N_0$	$k=0$	$0<k\leq 9$
$m=0$	$\frac{C_9^m \cdot 9^{10-m}}{9 \cdot 10^9} = \left(\frac{9}{10}\right)^{10-m}$ 10-digit number does not have a digit 0,	$\frac{C_9^m \cdot 8 \cdot 9^{9-m}}{9 \cdot 10^9} = \frac{8}{9} \cdot \left(\frac{9}{10}\right)^{9-m}$ 10-digit number does not have a digit k (k≠0),
$0<m<10$	$\frac{C_9^m \cdot 9^{10-m}}{9 \cdot 10^9}$ The digit 0 occurs exactly m times but 0 is not in the first position,	$\frac{C_9^{m-1} \cdot 9^{10-m} + C_9^m \cdot 8 \cdot 9^{9-m}}{9 \cdot 10^9}$ The digit k (k≠0) occurs exactly m times;
$m=10$	0 The digit 0 occurs exactly 10 times	$\frac{C_9^{m-1} \cdot 9^{10-m}}{9 \cdot 10^9} = \frac{1}{10^9}$ If the digit k (k≠0) occurs exactly 10 times then the number is $\overline{kkkkkkkkkk}$

> *Ex.3. An infinite sequence of independent trials is to be performed. Each trial results in success with probability p and a failure with probability 1 − p. What is the probability that (a) at least one success occurs in the first n trials; (b) exactly k successes occur in the first n trials;(c) all trials result in successes?* (Ross, 2010)

This exercise has three parameters with the following domains: an interval $[0;1]$ for a probabilistic primary parameter p, the set N for n and a subset of N_0 for k (both n and k are enumerative primary parameters). The answer to part (a) is $1-(1-p)^n$ where $(1-p)^n$ is the probability of the complementary event (no successes in the first n trials). The probability to part (b) for a Bernoulli trial if $n \geq k$ is given by $C_n^k \cdot p^k \cdot (1-p)^{n-k}$. To answer part (c), at first, the probability of the first n trials all resulting in success is found p^k. To calculate the limit $\lim_{n\to\infty} p^n$ the domain $[0;1]$ is divided into subcases. If p=1 then $\lim_{n\to\infty} p^n = 1$. If $p \in [0;1)$ then $\lim_{n\to\infty} p^n = 0$. Ex. 3 corresponds to algebraic reasoning level 5. The role of the parameter p can change when a problem solver uses different approaches: p is a placeholder for certain numerical values; p is a generalizer - when the general solution is constructed from solutions for specific p values; p is an unknown when the problem is to find out for which values of *p* the probability that all trials result in successes is 1. The parameter *p* as a changing quantity means that the problem solver recognises that for *p=1* and *p=0.4* the probability that all trials result in successes is different.

Ex.4. A participant in the lottery "Latloto 5 no 35" has sent two completed cards to the same lottery. Determine the probability that the participant will win two minimum prizes in the current draw (for each card exactly three numbers of the "Latloto 5 no 35" lottery results match).

Table 3 ***The answer of Ex.4*** *(created by the authors)*

$n \in N_0$	the number of favorable cases	the probability
n=0 or $n \geq 6$	0	0
1	$C_4^2 \cdot C_4^2 = 36$	0.0001
2	$C_3^1 \cdot C_3^1 \cdot C_{27}^1 + C_2^1 \cdot C_3^2 \cdot C_3^2 = 261$	0.0008
3	$C_3^1 + C_3^2 \cdot C_2^1 \cdot C_2^1 \cdot C_{28}^1 + C_{28}^2 = 717$	0.0022
4	$C_4^2 \cdot C_{29}^1 + C_4^3 \cdot C_{29}^2 = 1798$	0.0055
5	$C_5^3 \cdot C_{30}^2 = 4350$	0.0134

For a description of the "Latloto 5 no 35" lottery, see (Latvijas loto, 2022). Ex.4 has no primary parameters. $C_{35}^5 = 324632$ is the number of possible filling combinations. The probability of getting two minimum wins depends on the number of matching numbers in these two cards. Therefore, the hidden secondary parameter $n \in N_0$ is introduced for the number of matching numbers in two cards. If *n=0* or $n \geq 6$, then the probability of the participant receiving two minimum wins in the current draw is 0, but for other values, the number of favourable cases is calculated by different formulae given in Table 3. Ex. 4 corresponds to algebraic reasoning level 4. The hidden secondary parameter *n* is a placeholder for the cardinality of the matching numbers in these two cards. The parameter *n* has the role of a changing quantity when the solver realises that the answers for different values of *n* involve very different forms and no simple single formula is possible.

Ex.5. Find the probability of rolling an exact sum n out of the set of four six-sided fair dice (Sas, 2021).

This example demonstrates the solution process with an auxiliary secondary parameter. The sum n satisfies the global condition $n \in N$, but there are a number of non-zero cases only for $4 \leq n \leq 24$. Let U be the set of sequences of x_1, x_2, x_3, x_4 which satisfy the equation (1). The cardinality $|U| = C_{n-1}^3$.

$$x_1 + x_2 + x_3 + x_4 = n. \qquad (1)$$

where n – sum of four six-sided fair dice,

x_i- positive integer for each index $i \in \{1;2;3;4\}$

For each index $i \in \{1;2;3;4\}$ one introduces two sets of sequences x_1, x_2, x_3, x_4 which satisfy the equation (1), A_i additionally satisfies the inequality $x_i > 6$, but $\overline{A_i}$ - the inequality $x_i \leq 6$. Using the stated notions one must find the cardinality $|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}|$, see (Sondore & Daugulis, 2018).

Table 4 ***The answer of Ex.5*** *(created by the authors)*

$n \in N$	the probability of rolling an exact sum *n* out of the set of four six-sided fair dice
$n \leq 3$ and $25 \leq n$	0
$4 \leq n \leq 9$, k=0	$\frac{C_{n-1}^3}{1296}$
$10 \leq n \leq 15$, k=1	$\frac{C_{n-1}^3 - 4 \cdot C_{n-7}^3}{1296}$
$16 \leq n \leq 21$, k=2	$\frac{C_{n-1}^3 - 4 \cdot C_{n-7}^3 + 6 \cdot C_{n-13}^3}{1296}$
$22 \leq n \leq 24$, k=3	$\frac{C_{n-1}^3 - 4 \cdot C_{n-7}^3 + 6 \cdot C_{n-13}^3 - 4 \cdot C_{n-19}^3}{1296}$

In this step, an auxiliary secondary parameter k is introduced, k being the maximum number of roots (which are greater than 6) of equation (1). The aim of this step is to simplify the explanations of the problem-solving process.

Case *k=0* determines subdomain $4 \leq n \leq 9$ because it is not possible that some integer x_1, x_2, x_3, x_4 is greater than 6 but the sum $x_1 + x_2 + x_3 + x_4$ still belongs to the interval [4;9]. Therefore for each index $i \in \{1;2;3;4\}$ the cardinality of the set A_i is zero and $|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}| = |U| - 0 = C_{n-1}^3$.

Case *k=1*, the equation (1) may have at most one root $x_i > 6$ that determines subdomain $10 \leq n \leq 15$. The solution for placeholder n=10 is analysed in more detail below. At first the explanation of finding cardinality $|A_1|$ is given. For this set of sequences A_1 the root $x_1 > 6$. Assume that integer $z_1 = 6$, it is the maximal possible value for fair dice. Then $n - z_1 = 10 - 6 = 4$, the rest of the sum 4 is expressed as the sum of four terms $y_1 + y_2 + y_3 + y_4 = 4$, (where y_i are positive integers for each $i \in \{1;2;3;4\}$). The root x_1 of the equation (1) will be $x_1 = z_1 + y_1 > 6$ but each y_i is not greater than 6. $|A_1| = C_3^3 = C_{n-7}^3$. Any of the other roots x_i may be greater than 6, therefore the number of redundant possibilities for n=10 is $\sum_i |A_i| = 4 \cdot C_{n-7}^3$. The answer for other values of n within the range $10 \leq n \leq 15$ is calculated arguing similarly. Therefore

$$|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}| = |U| - \sum_i |A_i| = C_{n-1}^3 - 4 \cdot C_{n-7}^3.$$

The case $k=2$ determines a subdomain $16 \leq n \leq 21$ but the case $k=3$ determines a subdomain $22 \leq n \leq 24$. The solution for these cases is analysed in more detail in (Sondore & Daugulis, 2018). The number of all possible cases is 1296. The obtained formulae are summarized for six subdomains of the parameter n in Table 4. The general formula for probability is quite complex. Ex. 5 corresponds to algebraic reasoning level 6. In this problem, the auxiliary secondary parameter k determines the number of summands in the formula of the Inclusion-Exclusion principle. The parameter k is a placeholder and its role does not change during the solution process. The role of the enumerative primary parameter n changes during the problem-solution process (placeholder, changing quantity, and generalizer). Ex. 5 also provides a way to individualize the learning process by choosing different values for the parameter n for different students.

Conclusions

A considerable number of introductory probability problems with higher-order variables - parameters have been analysed. The authors have obtained a classification of the parameters for the probability problems, which have the following characteristics: determination of the number of combinations; calculation of simple and compound probabilities. The classification depending on the type of parameters in introductory probability problems is as follows:

- primary parameters: enumerative or probabilistic;
- secondary parameters: hidden or auxiliary.

When selecting an introductory PPP with the desired parameter types and algebraic reasoning levels, a teacher can design tasks with different difficulty levels by using different parameter values. In this way, the proposed parameter classification can be useful in differentiating and individualizing the college-level study of probability theory and statistics required in the current period of distance learning. PPP's provide experience and skills in partitioning parameter ranges and finding general formulae for subcases. Reviewing the answers to the solved PPP in the textbook provides the experience to check the permissible values (domain) of the parameters.

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